Parrini on the a priori in Logical Empiricism

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ABSTRACT

Logical empiricism defined itself, in part, by rejecting Kant's claims of knowledge, particularly his notion of *a priori*. Non-Euclidean geometries and related general relativity question the Euclidean ground of the notion, empirical evidence replaces any *a priori* grounding, and logical-mathematical truths signify tautologies incompatible with any *a priori* synthesis (Russell 1897; Wittgenstein 1922). Ultimately, scientific philosophy allows no room for the psychologistic mind-dependency. Kant's entire cognitive framework became untenable. Nevertheless, evaluating the notion of *a priori* in logical empiricism remains problematic. For Kant, *a priori* means 'necessary and unrevisable' but also 'constitutive and contingent' (Friedman 1999, 2007; Parrini 1998; De Boer 2010). A close analysis shows that neo-positivists transform rather than abandon Kant's notion by developing various proposals without consistency. Parrini (2002) groups this variety of readings into two types: a *weak* and a *strong* rejection of the Kantian *a priori*. He argues that only the weak rejection accurately describes the evolution of the *a priori* in logical empiricism. This paper aims to support Parrini's classification. Part I outlines Parrini's neo-positivist account of scientific knowledge. Part II analyzes the *a priori* in Kant's theory of judgments, discussing empirical *a priori* propositions (Kant 1787; Harper 1989) as well as the material *a priori* (Husserl 1900/1901, Schlick 1930, Simons 1992, Silva 2017). Part III criticizes Einstein's objections to Kantian intuition, significantly influencing Schlick's *a priori* conventionalism (Einstein 1919). Part IV assesses the critique of the *a priori* advocated by metric geometry and Russell, which define the epistemic background of the neo-positivists. Parts V through VII examine the developments of the *a priori* in logical empiricism, including the relativized *a priori* (Friedman 2001, 2009), implicit definition (Hilbert 1902; Schlick 1918), and coordinative principles (Reichenbach 1920; De Boer 2010). Final remarks compare these developments with Carnap's *a priori L-rules* (1928). provide the abstract

1. The Question

Since its outset, the logical-empiricist movement has sharply criticized Kant's analysis of knowledge (1787, 1786) based on the pure-empirical distinction of

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the epistemic conditions and the notion of *a priori* cognition, deriving from a dubious synthesis *a priori* allegedly granting apodictic certainty to our claims. Neopositivists (Hahn, Schlick, Neurath, Reichenbach, Carnap, and others) unanimously reject these assumptions. Nevertheless, a clear assessment of the Kantian notion of *a priori* remains problematic due to various interpretations aiming to revise or reject its properties. Although cognitively relevant, the notion also appeared untenable after the developments, especially in geometry (Bolyai and Lobachevsky, Riemann, Minkowski, Hilbert, among others) and physics (Lorentz and Einstein), revolutionizing scientific research and its epistemic method.

Kant instantiates the purely *a priori* part of knowledge with Euclid's geometry and Newton's laws of motion, i.e., "the entire spatiotemporal framework of Newtonian physics," says Friedman (1999: 59). So, "synthetic *a priori* judgments belonging to the pure part of scientific knowledge," he argues, "represent the *conditions of possibility* of the empirical part" (Ibid). Roughly put, logical empiricists abandon those references and Kant's related epistemic model. So, they rethink the notion of *a priori* accordingly but develop different proposals without consistency. Parrini (see 2002: 38-42) reduces this variety of readings to two kinds, a *weak* and a *strong* rejection of the Kantian *a priori* and defends the claim that only the first kind correctly defines its evolution in the logical-empiricist movement.

Here I present some central claims of Parrini and their related issues supporting his signature thesis, confirmed by Friedman (1999, 2001) and De Boer (2010), that the logical empiricists and their references (Poincaré, Frege, Russell, Wittgenstein, Frank, and others) develop the Kantian *a priori* radicalizing its constitutive features and dismissing its psychologistic and apodictic features.

Particularly, part I outlines Parrini's view of neo-positivist account of scientific knowledge. Part II presents his analysis of the *a priori* in Kant's theory of judgements, discussing empirical *a priori* propositions (Harper 1989) and the material *a priori* (Husserl 1900/1901, Schlick 1930, Simons 1992, Silva 2017). Part III criticizes Einstein's main reasons for rejecting the synthesis *a priori*, especially its cognitively supporting intuition – reasons that significantly influenced the logical-empiricist manifesto (1929) of Hahn, Neurath, and Carnap. Part IV assesses the critique of the Kantian *a priori* advanced by metric geometry and Russell, defining the epistemic background of the neo-positivists. Part V and VII examine the notions of relativized *a priori* (i.e., constitutive and revisable) and coordinative principles in Reichenbach (1920) and Friedman

(2001, 2009), including De Boer (2010). This evolution of the Kantian *a priori* explicitly proves Parrini's thesis of *weak rejection*. Finally, Part VI presents a case of *strong rejection* instantiated by Schlick's notion of implicit definition (1918) that, nevertheless, reconsiders instead of neglecting Kant's *a priori*. Therefore, it implicitly shows the centrality of this latter even in one of the most anti-Kantian epistemologies conceived by the neo-positivists.

2. About Logical Empiricism

Parrini's views of the *a priori* knowledge and its related issue (especially analyticity) strictly depend on his conception of logical empiricism, namely the epistemological tradition he defends in his writings (1998, 2002).

Parrini traces the origins of logical empiricism back to the Vienna Circle (i.e., Verein Ernst Mach) and the Berlin Circle (i.e., Gesellschaft für empirische Philosophie), led by Moritz Schlick and Hans Reichenbach, respectively. The two groups openly joined their efforts when Reichenbach and Carnap took charge of the journal *Annalen der Philosophie und philosophischen Kritik* in 1930 and renamed it *Erkenntnis*. Consistently, Parrini sees logical empiricism as an outcome of Mach's natural scientism[1] (Banks 2014; Pojman 2020), Russell's logical analysis, and, especially, Wittgenstein's *Tractatus* (1922). He groups their defining issues under two main claims *(a-b)* and their related lemmas *(c1-3)*.

a) All logical-mathematical truths have a tautological nature, including the *a priori* propositions.

b) The meaning of a statement is constituted by its method of empirical verification.

According to *(a)*, the so-called linguistic doctrine of the *a priori* establishes that cognitively significant statements can be divided into the two jointly exhaustive and mutually exclusive classes of analytic and synthetic statements. Thus, to hold *(a)* entails dismissing its opposing notions, namely Kant's synthetic *a priori* judgments and Husserl's material *a priori*. While *(b)* states the principle of verifi-

[1] See Parrini (2017).

cation. This latter, roughly put, says that logical empiricists agree on using evidence obtained through observation or experimentation *to prove or disprove a hypothesis*. Accordingly, experiential statements become the standard reference for judging (in general) not only the empirical correctness of a statement but also the meaningfulness of any statement whatsoever (see Parrini 2002: 13-4). The latter kind of statement is supposed to rely on the former. Still, logical empiricists disagree on the type of connection between these two kinds of assertions, the one constituting the empirical basis for the other representing all propositions endowed with meaning. All advanced proposals solely agree on including in the cognitive discourse all statements of a paradigmatically scientific nature and excluding those paradigmatically metaphysical in nature.

From *(a)* and *(b)*, Parrini derives three lemmas or theses *(c1-3)* and clarifies that only *(c1)* has a shared endorsement while *(c2)* and *(c3)* create discordances among the logical empiricists.

> *(c1)* Metaphysics is rejected as a cognitive discourse devoid of meaning and confineable among artistic productions expressing feelings.
>
> *(c2)* Philosophy is essentially an analysis of language.
>
> *(c3)* A coherent scientific conception of the world is achievable through the encyclopedic unification of the sciences.

As Parrini remarks, Schlick and Carnap disagree on *(c2)*. While *(c3)*, conceived by Neurath, is gradually set aside. Interestingly, *(c1)*'s second part connects logical empiricism to Nietzsche's thoughts (see Carnap 1931).

For Parrini, introducing these claims and lemmas as developments of Hume's empiricism would undermine their historical relevance and novelty. Logical empiricism is much more than a logical-linguistic variation of the XVII-XVII century empiricism, forgetful of past epistemic achievements (especially Kant's and Poincaré's). For this purpose, Parrini recalls the physicist Philipp Frank's and the mathematician Hans Hahn's contribution to the prehistory of the Vienna Circle around 1907. E.g., Schlick (1918) and Frank (1949) openly urge to overcome Kant's apriorism through Poincaré's conventionalism. The following passage entirely captures and validates Parrini's historical reconstruction.

> We felt very strongly that there was a certain gap between the description of observations, necessarily vague and complex, and the principles of science,

> consisting, in physics particularly, of a small number of concepts (like force, mass, etc.) linked by statements of great simplicity. We admitted that the gap between the description of facts and the general principles of science was not fully bridged by Mach, but we could not agree with Kant, who built this bridge by forms or patterns of experience that could not change with the advance of science. [...] In our opinion, the man who bridged the gap successfully was the French mathematician and philosopher Henri Poincaré. (Frank 1949: 8)

Frank's words show the significance of Poincaré for the emergent Vienna Circle, which relied on his conventionalism to overcome Mach's analysis of the psychophysical relations (1897) and Kant's rigidity of the cognitive *a priori* (1787). Still, Frank implicitly reveals that Parrini's *(b)* claim on verification as a standard for any linguistic meaning is central to the neo-positivist movement. Still, the logical-empiricist relationship with the Kantian *a priori* is uneasy. Parrini devotes a large part of his analysis to it.

3. The Kantian A Priori

Parrini (1994, 2002) identifies four features *(a-d)* representing the Kantian theory of judgments and its related issues about the *a priori*. He also offers some insight into the controversial kind of judgments that, although empirical, carry necessity *(b2)*, usually overlooked by scholars, and the incomprehensible distinction between judgments of perception and judgments of experience *(b3)*.

(a) A priori and A Posteriori

"The distinction between *a priori* and *a posteriori* judgments," clarifies Parrini, "has an epistemological nature and pertains to the foundation of the validity of judgments" (2002: 133). Further, "the validity of *a priori* judgments is independent of experience and is therefore universal and necessary" and "the validity of a posteriori judgments instead depends on experience and is therefore not universal and necessary" (Ibid). Following Kant (1787) and Ayer (1952), Parrini also specifies that *a priori* knowledge isn't independent of a particular experience but absolutely independent of any experience. In contrast, empirical knowledge is possible only *a posteriori*, i.e., through experience.

(b) Synthetic and Analytic

Besides *a priori* and *a posteriori*, Kant also defines judgments as synthetic and analytic, one of the most problematic characterizations for logical empiricists. For Parrini, "in analytic judgments what is stated in the predicate nothing adds to what is already contained in the subject," meaning that "in these analytic judgments, the 'characteristic marks' constituting the concept of the predicate are already included among the marks constituting the concept of the subject" (2002: 134). Such judgments carry three properties. They 1) don't amplify our previous knowledge, 2) are valid independently of experience, and 3) are therefore *a priori*.

On the contrary, synthetic judgments can broaden our knowledge. In them, the predicate attributes to the subject properties not already contained in the concept of the subject itself.

> As we know, what describes Kant's conception is the affirmation that these synthetic judgments can advance knowledge by leaning on experience or solely leveraging on the transcendental (epistemic) conditions of the knowing subjectivity. In the first case, we have a posteriori synthetic judgments whose validity depends precisely on experience and whose possibility does not seem problematic. In the second case, the [cognitive] increase instead occurs a priori, independently of experience, and we have a priori synthetic judgments. (Parrini 2002: 135)

Since it's unclear how synthetic judgments can rely on the *a priori* conditions of knowledge, the main objective of Kant's first *Critique* (1787) consists precisely in showing its possibility. In this case, the help from experience is entirely missing. So, if we combine concept *A* with concept *B* lying outside *A*, how can we justify their synthesis?

> What is the X here on which the understanding depends when it believes itself to discover beyond the concept of *A* a predicate that is foreign to it and that is yet connected with it? It cannot be experience, for the principle that has been adduced adds the latter representations to the former not only with greater generality than experience can provide, but also with the expression of necessity, hence entirely *a priori* and from mere concepts. (Kant 1787: A9/B13)

In his first example of synthetic *a priori* proposition, Kant says, "everything that happens has its cause" (A9/B13); the concept of *something that happens* (i.e., the *S*-term or *A*-concept) denotes an existence preceded by a time. "But the concept of a cause indicates something different from the concept of something that

happens," argues Kant, "and is not contained in the latter representation at all" (Ibid). The *S*-term or *A*-concept (i.e., *something that happens*) does not include the *P*-term or *B*-concept (i.e., *cause*). So, the connection of the terms in the judgment shows no containment. Despite that, we cognize the concept of *cause* as belonging to the concept of *what happens in general*, even though not contained in it. The concept of *cause* is foreign to the concept of *what happens in general* but is nevertheless connected with it and even necessarily connected. So, Kant asks what is "the unknown = *X*" justifying this connection.

As Parrini notices, the *a priori* synthetic judgments not only allegedly increase knowledge and do so necessarily, but they also transfer the feature of necessity and certainty to some factual judgments, which are necessary because of them. To question the possibility of such synthetic *a priori* judgments requires finding "the unknown X on which the intellect rests when," comments Parrini, "it believes in tracing out of the concept A the foreign predicate B, nevertheless connected to A in such a way as to produce universally necessary valid judgments about the objects of experience" (2002: 135). Roughly put, in Kant's transcendentalism, synthetic *a posteriori* judgments are problematic too. Indeed, "it is certainly clear that in their case, the *x* on which the intellect relies to broaden knowledge is represented by experience" (Ibid). However, "synthetic *a priori* judgments," argues Parrini, "are based on the conditions of the possibility of experience, and therefore on the conditions of possibility proper to synthetic *a posteriori* judgments" (Ibid). Parrini's words mean that synthetic *a priori* judgments condition certain synthetic *a posteriori* judgments and so bestow them necessity and certainty.

Hence, Kant identifies five kinds of judgments:

(1) the synthetic *a posteriori*, e.g., "all bodies are heavy" (A7/B11)
(2) the analytic *a priori*, e.g., "all bodies are extended" (A7/B11)
(3) the synthetic *a priori*, e.g., "7+5=12" (B15) and "space has three dimensions" (A239/B299)
(4) the empirical a priori, e.g., "every alteration has its cause" (B3)
(5) the judgment of perception, e.g., "I see the air and it feels elastic" (see AK 4: 399)

As he discusses his theory of judgments (see A6-10/B1-24; Hanna 2022), Kant mentions judgments combining *(1)* and *(2)*, which some scholars (see Harper 1989) name mixed judgments *(4)*. Further, he also fails to make sense of judg-

ments of perception *(5)*, likely a mistake, although worth confuting. Below, I briefly discuss *(4-5)*.

To confirm Parrini's insight on *(4)*, i.e., mixed judgments, I refer to Harper (1989). Indeed, Kant's wording needs clarification, are empirical *a priori* judgments possible?

> Among *a priori* cognitions, however, those are called pure with which nothing empirical is intermixed. Thus, e.g., the proposition "Every alteration has its cause" is an *a priori* proposition, only not pure, since alteration is a concept that can be drawn only from experience. (Kant 1787: B3)

Although the passage looks misleading, Kant can exclusively mean that some empirical judgments (precisely, the scientific) show apodictic certainty. Indeed, he argues that necessary cognitions, in the strictest sense universal, are purely *a priori* judgments. However, another example from "the commonest use of the understanding," he says, could be "the proposition that every alteration must have a cause" (B5). Unlike Hume, Kant holds not only that "the very concept of a cause so obviously contains the concept of a necessity of connection with an effect and a strict universality of rule" but also that "experience itself gets its certainty" from it (Ibid). Hume would only agree with the definition of causality, not with its applicability to empirical instances. On the contrary, for Kant, the contingency of experience merely suggests a non-empirical origin of necessity. However, it doesn't disprove "the reality of pure *a priori* principles in our cognition" and "their indispensability for the possibility of experience itself" (Ibid).

Harper furthers Parrini's insight. He notices that by default, Kant characterizes the notion of *a priori* knowledge with *(a)* inner (i.e., strict) necessity, conceived as coextensive with *(b)* independence of experience and *(c)* contrasting with empirical or *a posteriori* knowledge. But Harper argues that *(a)* doesn't require the dismissal of *(c)*; therefore, *(b)* results problematic. To exemplify this issue, he quotes Newton's *Principia* (1687: Proposition II, Book III).

> The forces by which the primary planets are continually drawn away from rectilinear motions and are maintained in their respective orbits are directed to the sun [...]. (Newton 1999: 802)

We cannot describe this kind of knowledge as "merely borrowed from experience," i.e., "cognized only *a posteriori*, or empirically" nor as "independently of experience" (A2), meaning *a priori*. For Harper, Newton's claims (including, perhaps, his principle of universal gravitation) "count for Kant as interesting examples of mixed items of knowledge inferred from a combination of experience

and strictly universal principles" (1986: 240). Further, and more importantly, according to the characterization of necessity discussed in the *Postulates of Empirical Thinking in General* (A218/B205), "such mixed items of knowledge count as necessary" (Ibid).

To complete the series of judgments, I briefly describe those of experience and perception. They represent two species of empirical judgment[2], first appearing in the *Prolegomena* (1783), then in the *Lectures on Logic* (1992)[3]. Here Kant lays the metaphysical foundations of natural science. The issue is that only judgments of experience supposedly involve the categories, entailing that those of perception don't. If so, this latter species is incompatible with the *B Deduction* of the first *Critique* (1787), where Kant argues (in contrast to the *Prolegomena*) that a judgment by default has objective validity and is, therefore, subject to the categories.

Reduced to its simplest terms, Kant questions how experience (i.e., objectively valid empirical knowledge consisting of judgments of experience) can arise from mere perception having only subjective validity. His solution (see AK 4: 298) points to the subsumption of the intuitively given content of perception under the categories, which takes place in two steps. First, the content is grasped by the merely *de facto* consciousness of one's particular mental state. At this step, we judge perceptually (whatever it may mean) without using the categories. Second, the same content is subsumed under the normative consciousness, where connections occur through categorically determined judgments (of experience).

Although inconsistent with his previous theory of judgments, Kant shockingly regards these steps as authoritative since they avoid the misunderstandings generated by the first *Critique* (1781).

> *Empirical judgments, insofar as they have objective validity*, are judgments of experience; those, however, that are *only subjectively valid* I call mere judgments of perception. The latter do not require a pure concept of the understanding, but only the logical connection of perceptions in a thinking subject. But the former always demand, in addition to the representations of sensory intuition, special *concepts originally generated in the understanding*, which are precisely what

[2] Allison disagrees with this claim. He instead holds that this couple pertains to two distinct conceptions of judgments (see Allison 2004: 179-82.

[3] In it, see *The Vienna Logic* (904), *The Hechsel Logic* (87), and *Dohna-Wundlacken Logic* (767).

make the judgment of experience *objectively valid.* (Kant 1783, AK 4: 298)

As Hatfield exemplifies, a merely subjective judgment (of perception) can be, "we see the sun shining on the stone and then the stone feels warm", while its corresponding universally valid judgment (of experience) can be, "the sun warms the stone" (2002: 39). Therefore, bestowing universal validity to a subjective judgment demands the employment of the categories. Nevertheless, the initial issue remains unsolved. In both editions of the first *Critique* (1781, 1787), a judgment's synthetic function is categorical by default. Therefore, it's unclear how connecting perceptions in a judgment (see AK 4: 3000) can occur without categorical synthesis.

(c) The Formal A Priori

Unlike Husserl and other phenomenologists, Kant holds that the *a priori* is strictly formal. Therefore, all the cognitive elements exclusively deal with the form of our knowledge. Among them are *(1)* the logical-analytical components of knowledge, depending on the chief principle of analytic judgments, namely that of non-contradiction, and *(2)* the logical-transcendental components constituting the subjective epistemic conditions of knowing (i.e., space, time, categories, schemata, and the principles of pure intellect).

Parrini clarifies that *(1)*'s elements abstract from any content of our knowledge and only consider the logical form shaping the relationship among cognitions, i.e., the form of thought in general. While about *(2)*, he explains that, although its elements have a synthetic nature, they are nevertheless formal since they represent the forms of sensibility and pure understanding.

Hence, Kant's formalism excludes the possibility of the material *a priori* by default since, concludes Parrini, "the *a priori* concerns the form of knowledge alone" (2002: 136). Following Schlick (1930), his critical target is Husserl's notion of material *a priori* (see Lanfredini 2006), which I briefly describe below.

For Husserl, *a priori* assertions don't require experience for validation, while *a posteriori* ones do. Like Kant, he maintains that necessity and universality characterize *a priori* assertions, which can be analytic or synthetic. However, Husserl also claims that certain synthetic *a priori* assertions can be empirical or factual, distancing himself from Kant. The defense of this claim is hard and starts by unfolding its implicit premises *(1-4)*.

(1) Analytic assertions are *a priori* by default. However, analyticity isn't defined

> by the containment of the predicate class in the subject class, and analytic truths aren't such only in virtue of the meaning of their constituting terms. For Husserl, instead, "analytic propositions are those whose logical value is preserved under formalization," where "to formalize is to substitute variables for names" (Silva 2017: 98).

In an analytic proposition it must be possible, without altering the proposition's logical form, to replace all material which has content, with an empty formal *Something*, and to eliminate every assertion of existence by giving all one's judgment the form of universal, unconditional laws.

It is, e.g., an analytic proposition that *the existence of this house includes that of its roof, its walls and its other parts*. For the *analytic* formula holds that the existence of a whole *W*(*A*, *B*, *C*...) generally includes that of its parts *A*, *B*, *C*... (Husserl 2001: 21)

> *(2)* For Kant, synthetic *a priori* truths show those structural or necessary properties of experience and its objects defining their *possibility*. For Husserl, "propositions whose *complete* formalization doesn't preserve logical value are synthetic" (Silva 2017: 98). Accordingly, "conceptual truths are *analytic*," explains Silva, "if the scopes of the concepts involved extent to the domain of *all* object[s]" (Ibid). However, restrictions may occur.

Some propositions, however, preserve truth-value under restrict formalization, that is, their logical value is preserved *provided* the variables are confined to more restrict domains of variability. If we think of these domains as extensions of concepts, synthetic truths are conceptual truths involving at least one concept whose scope is restricted to a proper subdomain of the domain of all objects [...]. (Silva 2017: 98)

> *(3)* Hence, *formal* concepts (e.g., number and whole) encompassing the totality of objects lack content, which *material* concepts acquire by delimiting their domain extension. So, particular domains are material by default, and related laws are synthetic. Consistently, material concepts are *a posteriori* if the conceptual truth they express is contingent but *a priori* if this truth is necessary.

> *(4)* As Simons notices, a proposition like "Nothing can be simultaneously red and green" isn't explicitly analytic for Husserl (1992: 374). Indeed, replacing 'green' with another property, such as 'round' or 'soft,' would result in a false statement. Neither it's plausible "to claim that the concept *red* includes the characteristic *not green*, so the propositions are not implicitly analytic either" (Simons 1992: 375).

Roughly put, Husserl argues that, given a part-whole (entirely) formal relationship, e.g., $P = (x, y)$, the specification of the parts introduces restrictions in it. For instance, if these parts (or variables) were further defined as *red* and *green*, their association under P (i.e., any class or something) would be prevented unless other restrictions intervened. E.g., if x and y stand for *(all) red* and *(all) green*, then $P = (x, y)$ at t^n is true only if n indicates any non-simultaneous t (i.e., time) determination. If correct, Husserl's notion of material *a priori* could denote a particular formalization restricted by factual implications, designating a regional field of special apriority. Consistently, Husserl conceives a variety of different material *a priori* ontologies.

Logical empiricists should welcome a factual feature affecting a proposition's apriority. However, Schlick unhesitatingly rejects it. He especially attacks Husserl's notion of 'intuition of essence' (Wesensschau), guilty of leading to "propositions of absolute validity which nevertheless have something to say concerning the stuff or material of experience" (1930: 278). Against Husserl, Schlick holds that an analytic proposition is true by virtue of its form alone. E.g., a tautology's meaning is *a priori* graspable. However, "in the case of a synthetic proposition," argues Schlick, "one must first understand its meaning, and afterwards [i.e., a posteriori] determine whether it is true or false" (1930: 279).

Influenced by Wittgenstein, Schlick believes that "a tautology is naturally an *a priori* truth, but gives expression to no state of affairs, and the validity of a tautology rests in no way upon experience" (1930: 281). Consequentially, Kant too was mistaken about synthetic *a priori* judgments. As a matter of principle, Parrini reminds us that, for Schlick's empiricism, "all propositions are either synthetic *a posteriori* or tautologous; synthetic *a priori* propositions seem to it to be a logical impossibility" (Ibid).

About sentences such as "every tone has a determinate pitch" or "that green spot isn't also red" (Schlick 1930: 282), Schlick concedes their apriority but opposes to their formality. So, although they genuinely provide *a priori* knowledge, they also have factual content characterizing their nature as material. Indeed, they deal with colors and sounds whose origin is experiential. Similarly, Kant differentiates between a cognition preceding (i.e., *prior to*) another and a cognition fully (or absolutely) *a priori* (see B24-5).

> So one says of someone who undermined the foundation of his house that he could have known *a priori* that it would collapse, i.e., he need not have waited for the experience of it actually collapsing. Yet he could not have known this entirely *a priori*. [...] we will understand by *a priori* cognitions not those that occur

> independently of this or that experience, but rather those that occur *absolutely* independently of all experience. Opposed to them are empirical cognitions, or those that are possible only *a posteriori*, i.e., through experience. (Kant 1787: B2-3)

Although Schlick relies on Kant's claims, he refers to Wittgenstein (1922, 1929) for disputing the notion of factual *a priori*, whose issue isn't cognitive but purely logical-semantical. Indeed, Husserl's mistake arises from a misunderstanding that the concepts of colors (or spaces) have "a formal structure" that "determines their meaning without remainder" (Schlick 1930: 284).

> Red and green are incompatible, not because I happen never to have observed such a joint appearance, but because the sentence "This spot is both red and green" is a meaningless combination of words. The logical rules which underline our employment of color-words forbid such a usage, just as they would forbid us to say, "Light red is redder than dark red." (Schlick 1930: 284)

Therefore, "it belongs to the logical grammar of color words that a word of this kind designates a specific property," concludes Schlick (1930: 285). So, these words "say nothing about existence, or about the nature of anything, but rather only exhibit the content of our concepts" (Ibid), ruling our linguistic employment of them. For Schlick, "Given the meaning of the words, they are *a priori*, but purely formal-tautological, as indeed are all other *a priori* propositions" (Ibid). On the contrary, Husserl maintains that the rules governing the meaningful use of linguistic terms aren't purely conventional but necessary, reflecting "essential semantic legalities related to the things these terms denote" (Silva 2017: 96). Husserl's synthetic *a priori* truths express these legalities pertaining to material essences and retain a cognitive value, denied by Schlick (see Piazza 2004).

(d) The Objective Validity of Subjective Conditions

As the last feature of the Kantian notion of *a priori*, Parrini points to the problematic "link between the *a priori* formal conditions of knowledge (i.e., the transcendental subject) and the level of the (empirical) knowing subjectivity" (2002: 136). Indeed, Kant holds that *(1)* "the formal conditions are also subjective" (Ibid). Nevertheless, this assumption challenges his criticism, which struggles to justify how such subjective conditions of thought can have objective validity and legitimately constitute the conditions of the possibility of any knowledge of ob-

jects. So, Kant questions "what is lawful (*quid juris*)" about our cognitive conditions, and not only "that which concerns the fact (*quid facti*)" that we have cognitions (84A/B116). Kant raises this distinction in his transcendental deduction, which isn't a deductive argument, but an argument endeavoring to "justify a right to possess and use something" (Allison 2004: 10). What is the validation of our pure concepts or categories? Here, Kant demands a (transcendental, i.e., non-empirical and more than logical) proof for possessing and employing them legitimately.

Correctly, Parrini relates this issue to another controversial claim of Kant, stating that *(2)* cognitive and ontological conditions overlap.

> The conditions of the *possibility of experience* in general are at the same time conditions of the *possibility of the objects of experience*, and on this account have objective validity in a synthetic judgment *a priori*. (Kant 1787: A158/B197)

Notwithstanding, *(2)* doesn't naturally derive from *(1)*. A close look reveals that *(1)* states that all cognitive conditions of objectivity are subjective (although strictly formal and never psychologistic), while *(2)* claims that these cognitive conditions are at the same time existential, meaning that they don't merely allow us to know a mind-independent object but determine its existence. Indeed, for Kant, the object of cognition is mind-dependent, cognitively and ontologically. More precisely, a concept is valid for an object only if the logical conditions it represents work on the existential conditions (provided by sensible intuition). Here, I set aside the controversies about the related questions of virtual objects (see Van Cleve 1999, Allison 2004), idealism (see Strawson 1966; Guyer 1987; Abela 2002; Allais 2015), and ignorance of things in themselves (see Langton 1998; Westphal 2004, 2021), debated by the scholars since the Feder-Garve's review (1782) that prompted Kant to clarify his epistemic claims in 1787.

4. Rejecting the Kantian A Priori

Notoriously, the logical empiricists reject Kant's notion of *a priori*. Like Schlick (1917), Parrini, for instance, doubts that "Kant really managed to purify the subjective conditions of knowing from every anthropological and psychologistic contamination and to effectively detach the dimension of the transcendental subjectivity, i.e., the place of the *a priori* forms of knowledge, from that of empirical subjectivity" (2002: 137). Still, Parrini also reminds us of some instances of agreement with Kant. E.g., the neo-positivists *(1)* accept the notion of formal-

ism; *(2)* Carnap employs the *a priori - a posteriori* distinction to understand the gap between formal sciences (such as mathematics and logic) and natural sciences (such as physics, chemistry, biology, etc.) and that between analytic and synthetic; *(3)* Schlick defends Kant's notion of *formal a priori* against Husserl's notion of *material a priori*.

Nevertheless, differences remain and mark the birth of the logical-empiricist movement. Parrini especially emphasizes the neo-positivist reduction of the *a priori* to analytic propositions. "Analytic propositions," he says, "exhaust the domain of the *prior*, unlike Kant believes" (2002: 139). Indeed, analytic or *a priori* propositions are tautologies, namely "true assertion by virtue of their 'pure form' alone, past any psychological consideration concerning the concrete processes of thinking" (Ibid). Further, the movement finds its identity in the shared rejection of the synthetic *a priori* judgments dominating Kant's epistemology. Although the *a priori* continues to coincide with the formal, it ceases to split into the two subclasses of the analytic and the synthetic, as Kant mistakenly believes. So, "the *a priori* coincides with the analytic and the necessary; and the *a posteriori* with the synthetic and the contingent," says Parrini (Ibid).

For this purpose, Parrini recalls the neo-positivist manifesto[4] signed by Hahn, Neurath, and Carnap in 1929.

> [...] logical analysis overcomes not only metaphysics in the proper, classical sense of the word, especially scholastic metaphysics and that of the systems of German idealism, but also the hidden metaphysics of Kantian and modern *apriorism*. The scientific world-conception knows no unconditionally valid knowledge derived from pure reason, no 'synthetic judgments *a priori*' of the kind that lie at the basis of Kantian epistemology and even more of all pre- and post-Kantian ontology and metaphysics. [...] It is precisely in the rejection of the possibility of synthetic knowledge *a priori* that the basic thesis of modern empiricism lies. The scientific world-conception knows only empirical statements about things of all kinds, and analytic statements of logic and mathematics. (Neurath 1973: 308)

However, what reasons precisely motivate rejecting Kant's synthetic *a priori* judgments? Parrini points to *(1)* the developments of the logical-mathematical sciences and the discussions on the foundations of logic, geometry, and arithmetic between the end of the XIX century and the first decades of the XX century. In particular, he refers to the works of Boole, Bolzano, Frege, Hilbert, Peano, Rus-

[4] Titled "Wissenschaftliche Weltauffassung: Der Wiener Kreis".

sell, and Wittgenstein. Among their results, Parrini mentions the symbolic-mathematical logic, the reduction of mathematics to logic (logicism), the formal-axiomatic foundation of geometry, the linguistic-tautological character of the truths in the propositional, the critique the psychologism (partially shared with Husserl's phenomenology), and linguistic conception of the *a priori*.

For Parrini, through *(1)*, the logical empiricists elaborate two central tenets.

> First, they [the neo-positivists] argued that the logical-linguistic relations connecting the statements among themselves have a purely formal character (i.e., a syntactic and semantics nature) and have nothing to do with the existing relationships among concrete thought processes. Second, they stated that some assertions (i.e., judgments) that Kant held to be synthetic *a priori*, namely the principles of pure mathematics, should actually be considered analytic and merged with those of deductive logic. (Parrini 2002: 141)

Through the second tenet, the neo-positivists explain the apodictic certainty of mathematics, the weakest issue for any empiricism, without conceding a genuinely informative and cognitive value to judgments devoid of dependence on experience (see Carnap 1930).

(a) Einstein and Kant on Mathematics

For Parrini, the second reason *(2)* for rejecting Kant's synthetic *a priori* derives from the developments of the physical sciences, *in primis* Einstein's theory of general relativity and quantum theory. They persuaded the neo-positivists to read as methodological prerequisites (i.e., not subject to truth or falsehood) or as *a posteriori* judgments some principles, such as Euclid's geometry or causality, which Kant considered synthetic *a priori*. For the neo-positivists, "these judgments'" says Parrini, "are *a priori* but not synthetic (and therefore analytic) and others synthetic but not *a priori* (and therefore synthetic *a posteriori*)" (2002: 142).

The support for these claims comes from Einstein's famous saying, "As far as the propositions of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality" (1954: 233). Still, Einstein is baffled that mathematics, a product of human thought independent of experience, can be so admirably appropriate to the objects of reality. However, he maintains that it's not the case for the "human reason without experience, merely by taking thought, [to be] able to fathom the properties of real things" (Ibid).

From Parrini's analysis, rejecting the synthetic *a priori* doesn't entail an overall dismissal of Kant's cognitive claims. E.g., Einstein's motivations are consistent with these claims, perhaps beyond his awareness. He holds that today axiomatics neatly separates "the logical-formal from its objective or intuitive content; according to axiomatics, the logical-formal alone forms the subject matter of mathematics" (1954: 233). So, it disregards any content associated with the logical-formal, which precisely corresponds to the Kantian notion of *pure* characterizing all cognitive conditions *a priori*, including their objects. Roughly put, the conditions enabling *a priori* knowledge, i.e., Kant's primary epistemic objective, must be pure or sensation-free.

> Every cognition is called *pure*, however, that is not mixed with anything foreign to it. But a cognition is called absolutely pure, in particular, in which no experience or sensation at all is mixed in, and that is thus fully *a priori*. (Kant 1787: A11/B24)

Also, as he establishes rules and boundaries of *a priori* cognition, Kant explicitly refers to "the objects of the pure understanding" (Axvi). Consistently, pure understanding deals with pure objects, entirely possible *a priori*. Kant identifies instances of *a priori* objects in mathematics and physics, "two theoretical cognitions of reason that are supposed to determine their objects *a priori*" (Ibid), the former entirely purely, the latter partially purely. In them, our "[pure] reason has to do only with itself" (Bx).

The consistency between Einstein's and Kant's views extends beyond Parrini's insight. For instance, as he considers the axioms of geometry, Einstein develops Kant-like arguments. Similarities hold even though the Riemannian has replaced Euclidean geometry instantiating Kant's mathematical claims (see A24/B39). Although these two kinds of geometries and their differences aren't my current goal, Einstein targets Euclid's definition of parallelism about the fifth postulate[5], the one adopted by Kant and disproved by Riemann.

[5] In Euclid's *Elements* (Book I, Postulate 5), the parallel postulate is an axiom stating, in two-dimensional geometry: "That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles." This postulate doesn't address parallel lines directly, but it involves parallelism. Indeed, the definition of parallel lines (Book I, Def 23) shortly precedes the five postulates.

> Parallel straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, don't meet one another in either direction. (Euclid's *Elements*, Book I, Def 23)

As Einstein reads it, "through two points in space there always passes one and only one straight line" (1954: 234). This axiom has been considered *a priori* since self-evident on the ground that "everyone knows what a straight line is, and what a point is" (Ibid). However, such an epistemic ground changes after Riemann.

> No knowledge or intuition of these objects is assumed but only the validity of the axioms, [...] which are to be taken in a purely formal sense, i.e., as void of all content of intuition or experience. These axioms are free creations of the human mind. All other propositions of geometry are logical inferences from the axioms (which are to be taken in the nominalistic sense only). (Einstein 1954: 234)

As Parrini discerns, Einstein shares some Kantian assumptions. *(a)* Axioms or (by adopting Kant's wording) pure mathematical claims are non-empirical; *(b)* on the contrary, mathematical axioms and their objects are mind-dependent. More importantly, *(c)* for Einstein, Kant-like *a priori* conditions entirely determine mathematical objects. As he argues, "The axioms *define* the objects of which geometry treats" (1954: 234). However, Einstein sees these conditions as aptly characterized by Schlick's nominalist notion of "implicit definition" (see Schlick 1974), departing from Kant. Accordingly, he also denies mathematics the capacity to "predicate anything about objects of our intuition or real objects" since, in axiomatic geometry, "the words 'point,' 'straight line,' etc., stand only for empty conceptual schemata" (1953: 234) whose content is mathematically irrelevant.

(b) Schlick's Notion of Implicit Definitions

However, are Schlick's implicit definitions entirely inconsistent with Kant's notion of cognitive object? Much depends on how Schlick (and Einstein) read 'sensible intuition' in Kant's criticism. If he refers to empirical intuitions (carrying a sensation), the formality and conventionality of an implicit definition deny any comparison. In contrast, pure intuitions don't grasp any object but form (i.e., formally structure) any object of cognition. In this latter sense alone, Kant's notion of intuition could be consistent with an implicit definition.

Consider, for instance, the axiom of extensionality in Zermelo-Fraenkel set theory: if X and Y have the same elements, then $X = Y$. As Jech (2003) formalizes it,

$$\forall u\,(u \in X \Leftrightarrow u \in Y) \Rightarrow X = Y$$

We can read it as follows, if X and Y are sets[6] such that for every object u, $u \in X$ iff (i.e., if and only if) $u \in Y$, then $X = Y$. Indeed, given the converse, namely, if $X = Y$ then $u \in X \equiv u \in Y$, which is "an axiom of predicate calculus" (Jech 2003: 6), we obtain

$$X = Y \text{ iff } \forall u\,(u \in X \Leftrightarrow u \in Y)$$

Suppose we dismiss the logical syntax that includes connectives (i.e., $\Leftrightarrow$ and $\Rightarrow$), membership and quantification (i.e., $\in$ and $\forall$, respectively). In that case, something remains, namely, the atomic formulas (i.e., X and Y) and their elements (i.e., u). Specifically, u represents a variable, not a concept. It doesn't group the common marks of several objects under itself, but it stands for those objects by containing them within itself as their placeholder. How do we grasp it?

Kant defines a sensible intuition "as containing an infinite number of representations within itself," while a concept "as a representation which is contained in an infinite number of different possible representations (as their common character), and which therefore contains these under itself" (A25/B40)[7]. Accordingly, he contrasts mathematical concepts to the philosophical and intends them as an alternative way of conceptualizing: "philosophical knowledge considers the particular only in the universal, mathematical knowledge the universal in the particular, or even in the single instance, though still always *a priori* and by means of reason" (A714/B742). Thus, "philosophical knowledge [...] has always to consider the universal *in abstracto* (by means of concepts), mathematics can consider the universal in concreto (in the single intuition) and yet at the same time through pure *a priori* representation" (A734-5/B762-3)[8].

[6] Differently said, if two sets have exactly the same members, then they are equal (i.e., they are one and the same set). It also means that A and B are the same object. If $A = B$, then logically anything that is true of the object A is also true of the object B (it being the same object). E.g., if $A = B$, then it is automatically true that for any object u, $u \in A$ iff $u \in B$ (this is the converse to the principle of extensionality). As usual, we write $A = B$ to mean that it is not true that $A \neq B$.

[7] Peruzzi calls these conditions as "invariant across domains" (2006: 452).

[8] For a logical reading of Kant's sensible intuitions, see Beth 1956, Hintikka 1967, Parsons 1969, Friedman 1990, Oliva 2018.

Hence, Einstein's criticism of the Kantian notion of sensible intuition could be hasty.

5. The epistemic background of the Rejection

From above, it's clear that neo-positivists radically revise or reject Kant's notion of *a priori*. However, Parrini warns us to consider the context of this rejection, especially two related aspects, namely *(1)* the relation between the negation of the synthetic *a priori* and the developments in the empirical sciences, and *(2)* the critique of transcendental epistemology. Indeed, they could better clarify the reasons justifying the neo-positivist detachment from Kant.

Parrini mainly supports *(1)* and *(2)* by referring to Russell (1897) and Reichenbach (1920).

(a) The A Priori and the Positive Sciences

Russell precisely unfolds the intricate relations between subjectivity and *a priori* as a cognitive form. Indeed, Parrini holds that Frege's logicism, the linguistic foundation of logical truths, and the critique of psychologism don't suffice "to free the *a priori* and the formal from any subjectivist contamination (i.e., the empirical subject), including any psychologist and anthropologist materialism" (2002: 143). In addition, it's also necessary to consider "the construction of alternative geometric systems and subsequent discussions about the possibility of an exclusively empiricist foundation of metric geometry" (Ibid). But what is metric geometry?

The chief objective of metric geometry is *metric spaces*, mostly applied to Riemannian geometry, but also group theory. In mathematics, a metric space is a set conceived together with the notion of *distance* between its elements (or points), measured by a (metric or distance) function. The basic idea is that any plane representing (or consisting of) a set of points can be equipped with different metrics.

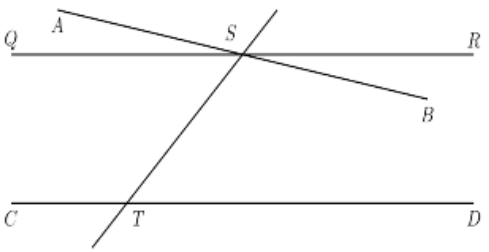

Figure 1

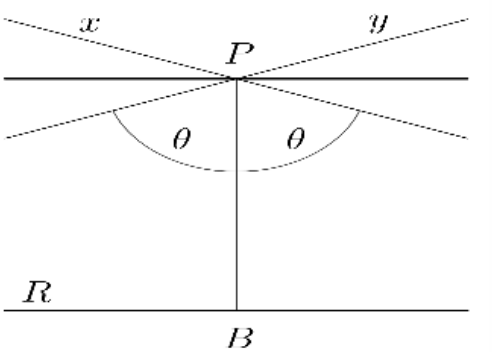

Figure 2

An interesting example of matric space is the hyperbolic plane derived from Bolyai's and Lobachevsky's geometry. Consider Euclid's Fifth postulate. Several attempts to replace it were made through the centuries. Geometricians used substitute postulates, equivalent to the Fifth, but simpler in statement. Among others, the one commonly adopted is Playfair's, although first stated as early as the fifth century by Proclus. It says, "through a given point can be drawn *only one* parallel to a given line." (Fig. 1) The Fifth implies Playfair's axiom, which institutively displays the difference with hyperbolic planes. Here, "For any given line R and point P not on R, in the plane containing both line R and point P there are *at least two* distinct lines through P that don't intersect R" (Fig. 2). By assuming that Playfair's axiom is false, we derive a non-Euclidean plane where entities, such as Saccheri quadrilaterals, exist. However, it also "means our triangle angle sums are strictly less than π," and that those quadrilaterals aren't rectangles because rectangles don't exist" (Harvey 2015: 411). In this case, summit angles are acute or obtuse but not right. Among other absurdities, the following theo-

rem holds: "Let *L* be a line and let *P* be a point that is not on *L*. Then there are *infinitely many* lines through *P* that are parallel to *L*" (Harvey 2015: 412), invalidating the Fifth.

For Parrini, the empirical determination of metric geometry has significantly contributed to refusing Kant's subjectivism. He substantiates his claims by recalling Russell's early work on the foundations of geometry (1897), where the Kantian *a priori* identifies with "the logical assumptions lying at the ground of the various branches of empirical knowledge, namely the assumptions of measurement" (2002: 143), showing the empirical character of the metric space. The *a priori* "is thus released from any reference to any psychological conditioning of the knowing subject" (Ibid).

Parrini is correct. As Russell clarifies what knowledge can be considered *a priori*, he dismisses its mental cause or genesis. He instead sees knowledge as "a datum to be analysed and classified" (1897: §3). Its analysis leads to "a formal and a material element in knowledge" (Ibid).

> The formal element will consist of postulates which are required to make knowledge possible at all, and of all that can be deduced from these postulates; the material element, on the other hand, will consist of all that comes to fill in the form given by the formal postulates – all that is contingent or dependent on experience, all that might have been otherwise without rendering knowledge impossible. We shall then call the formal element *a priori*, the material element empirical. (Russell 1897: §3)

Here Russell argues for an objective variation of the Kantian *a priori*, solely depending on the logical analysis of knowledge and dismissing subjective characterizations. The elements of a cognitive state supervening the *a priori* conditions of possible experience are subject to psychological investigation alone. Indeed, it's dangerous to leave the a priori "at the mercy of empirical psychology" (1897: §4) as Kant did, relying on pure intuition. Accordingly, the word '*a priori*' should be used without psychological implications. So, Russell suggests a purely logical test of apriority and asks, "Would experience be impossible, if a certain axiom or postulate were denied?" (1897: §5). Regarding a particular science, a more restricted o relative apriority could similarly be sorted out by asking what axiom or postulate would affect its subject matter.

Besides its psychological characterization, Russell also rejects the absolute apodicticity of the Kantian *a priori*. Indeed, "modern logic has shown that necessary propositions are always, in one aspect at least, hypothetical" (1897:

§6). If the *a priori* must express necessity, one must supply the ground or hypothesis on which alone the necessity holds.

> [However] this ground will vary from one science to another, and even, with the progress of knowledge, in the same science at different times. For as knowledge becomes more developed and articulate, more and more necessary connections are perceived, and the merely categorical truths, though they remain the foundation of apodeictic judgments, diminish in relative number. (Russell 1897: §6)

So, for Russell, the apodicticity of the *a priori* is relative at best, never absolute. However, as Parrini notices, Russell retains aspects of Kant's epistemology, for he endorses its epistemic method. "There are two grounds," states Russell, "on which necessity may be sought within any science," and they "may be (very roughly) distinguished as the ground which Kant seeks in the *Prolegomena*, and that which he seeks in the *Pure Reason*" (1897: §7).

> [1] We may start from the existence of our science as a fact, and analyze the reasoning employed with a view to discovering the fundamental postulate on which its logical possibility depends; in this case, the postulate, and all which follows from it alone, will be a priori. [2] Or we may accept the existence of the subject-matter of our science as our basis of fact, and deduce dogmatically whatever principles we can from the essential nature of this subject-matter. (Russell (1897: §7)

In this latter case, "it's that element in the subject-matter which makes possible the branch of experience dealt with by the science in question" (Russell 1897: §7). For Russell, these two grounds of necessity supplement each other. Still, "the first, as starting from the actual science, is the safest and easiest method of investigation, though the second seems the more convincing for exposition" (1897: §8), argues Russell. However, though their methods of investigation differ, their results line up. "For in the first case," says Russell, "by analysis of the science, we discover the postulate on which alone its reasonings are possible" (Ibid). As Kant rightly assumes, "if reasoning in the science is impossible without some postulate, this postulate must be essential to experience of the subject-matter of the science, and thus we get the second ground" (Ibid). Such a conclusion confirms Parrini's initial insight.

Notwithstanding, Parrini notices a tension between Russell's Kant-like view of the *a priori* and Reichenbach's. He even calls it a controversy centered on the notion of pure intuition, which nearly anticipates Reichenbach's "famous dis-

tinction between 'context of discovery' and 'context of justification' as well as Carnap's identification of epistemological research and the logical analysis of scientific language" (Parrini 2002: 144). Still, for Parrini, what matters here is the shared objective among logical empiricists and early analytic philosophers, namely the radical rejection of the psychologistic traits mystifying the Kantian notion of *a priori*.

> The results discovered by the positive sciences in continuous contact with experience presuppose principles the detection of which by means of logical analysis the task of philosophy. [...] It must be realized that there is no other method for epistemology *than to discover the principles actually employed in knowledge*. Kant's attempt to detect these principles in reason must be regarded as a failure; an inductive method must replace his deductive method. The method is inductive insofar as it is tied to the actual empirical data. Of course, the analytic method as such is not equivalent to inductive inference. In order to avoid confusion we shall call it the method of logical analysis. (Reichenbach 1965: 74-5)

(2) The Elimination of Transcendental Epistemology

Notwithstanding, for Parrini, fully appreciating the neo-positivist rejection of Kant's synthetic *a priori* requires understanding the context in which the empirical sciences developed. Their advancement is irreducible to the achievements of relativistic physics (i.e., the mechanics compatible with special relativity and general relativity) and quantum physics, which showed the empiricist or *a posteriori* synthetic character of principles that Kant believed to be synthetic *a priori* (such as the Euclidean planes or the principle of causality). So, Parrini argues that the most revolutionary aspect of scientific developments pertains to the kind of dependence on experience.

Since Galileo and Newton, "physical sciences have come to progressively assume an abstract theoretical aspect so that, while maintaining a fundamental relationship with experience, they have ended up resembling increasingly mathematical theories" (Parrini 2002: 147). So, Parrini recalls the discussions on the empirical or conventional foundations of certain physical principles and their presentation in deductive-axiomatic (*more geometrico*) way geometric of some areas of physics, starting from mechanics. E.g., Hertz (1894) emphasizes the symbolic-abstract component of the theories. He argues that to deduce the future from the past and obtain the desired predictions, we form subjective and in-

ternal images (innere Scheinbilder) or symbols of external objects precisely in such a way that the logical implications of our images mirror the necessary implications that the represented objects display in nature. Similarly, Poincaré (1902) discusses the issue of whether mechanics has a conventional or empirical nature, the former endorsed by the "continentals" and the latter by the "Britons." Is mechanics an experimental or a deductive and *a priori* science? Poincaré's question exemplarily presents Parrini's issue, including the answer.

> [Finally,] it's impossible to understand the nature of mechanics if we neglect the presence in it of elements of a conventional nature and persevere in writing treatises in which we don't distinguish 'very neatly what experience and what mathematical reasoning is, what convention and what hypothesis is.' (Parrini 2002: 147)

For Parrini, the *a priori* component of scientific knowledge dismisses any transcendental characterization. It instead derives from empirical observation. E.g., in recognizing the constructive nature of physical theories, Einstein claims that understanding a group of natural processes invariably means that we have found a constructive theory that covers the processes in question. However, another theory underpins this construction.

> Along with this most important class of theories [i.e., the constructive ones] there exists a second, which I will call "principle-theories." These employ the analytic, not the synthetic, method. The elements which form their basis and starting-point are not hypothetically constructed but empirically discovered ones, general characteristics of natural processes, principles that give rise to mathematically formulated criteria which the separate processes or the theoretical representations of them have to satisfy. (Einstein 1954: 228)

Therefore, Einstein advocates for an *a priori* of empirical origin, proving correct Parrini's claims. Accordingly, the neo-positivist criticism of the Kantian notion of synthetic *a priori* is ultimately rooted in the theory of general relativity for two reasons. First, this latter revises the classic notions of space, time, and movement, including their Euclidean foundations. Second, and more significantly, it also entails a theoretical-abstract reconstruction of those notions. Indeed, Einstein's general relativity requires definitions or conventions established *a priori* in a non-transcendental sense.

6. The Relativized a Priori

Although unforeseen revolutionary changes within the exact sciences historically jeopardized the Kantian conception of *a priori* principles, Friedman (2001, 2009), like Parrini opposes a radical rejection of Kant. Building on Reichenbach (1920), he articulates the notion of a dynamical or relativized *a priori* "within a historical account of the conceptual evolution of the sciences rather than a purely syntactic or semantic account of the formal language(s) of the sciences" (2001: xii). For him, exact sciences don't rely on Quine's holism but on "the notion of a relativized yet still constitutive *a priori*" (2001: 71), although "*a priori* principles (both mathematical and physical) change and develop with the continual progress of empirical natural science" (Ibid), unlike Kant thought. Nevertheless, if they evolve according to empirical findings, in what sense are they still Kantian?

> According to the traditional conception of the a priori, in which it means "justified independently of experience," [...] any principle correctly characterized as a priori would perforce have to hold (if it does hold) entirely independently of all empirical findings and would thus have to hold "come what may." (Friedman 2001: 71)

A priori principle must be unrevisable by default. Still, "this type of unrevisability or independence from experience applies equally to all principles of natural science, including," comments Friedman, the "empirical laws" (2001: 72). Notwithstanding, Friedman (2007) follows the logical empiricists and distinguishes two meanings of the Kantian *a priori*. Accordingly, this latter is *(a)* "necessary and unrevisable, true for all time," and *(b)* "constitutive of the concept of the object of [scientific] knowledge" (Ibid). The distinction appears first in Reichenbach (1920)[9], then in Carnap (1928), but it's prefigured in Kant. His *a priori* principles are *(a)* absolutely (i.e., non-relatively) necessary or apodictically certain, but they also *(b)* perform a "constitutive function with respect to *a posteriori* or empirical truths," says Friedman, (2001: 73). Accordingly, this function makes the empirical cognition of those truths first possible, including their empirical confirmation.

The reason that *a priori* knowledge is in fact independent of empirical cognition

[9] As Reichenbach says, "Kant's concept of *a priori* has two different meanings. First, it means "necessarily true" or "true for all times," and secondly, 'constituting the concept of object'" (1965: 48)

> or experience, for *Kant*, is that a priori knowledge yields the necessary conditions under which alone empirical cognition or experience can take place. Since they formulate the necessary conditions or rules for establishing empirical knowledge, *a priori* principles cannot themselves be similarly established; and it is in precisely this sense that they are prior to or independent of experience. (Friedman 2001: 73)

However, once we acknowledge that "those principles Kant took to be *a priori* can after all be revised," says Friedman, "the way is then open, as it was for Reichenbach and Carnap, to retain Kant's characteristic understanding of *a priori* principles as constitutive" (2001: 73) while dismissing the marks of necessity, unrevisability, and apodictic certainty.

Essential to this notion of relativized *a priori* (i.e., constitutive and revisable) is the relation between *a priori* and *a posteriori* cognition. What exactly does it mean for the *a priori* to represent *(a) necessary conditions* and *(b) constitutive principles* of empirical knowledge, especially considering the post-Kantian revolutionary developments of sciences? Kant overlaps *(a)* and *(b)*, holding that constitutive principles are necessary conditions (of the possibility of empirical laws). Still, Friedman warns us that *(a)* differs from a standard sense, "where A is a necessary condition of B simply if B implies A" (2001: 74). Indeed, in the Kantian sense, *(a)* entails *(b)*, namely a normative framework for empirical knowledge.

> To say that A is a constitutive condition of B rather means that A is a necessary condition, not simply of the truth of B, but of B's meaningfulness or possession of a truth value. It means [...] that A is a *presupposition* of B. (Friedman 2001: 74)

E.g., in Newtonian physics, the law of universal gravitation essentially employs the concept of absolute acceleration that has no empirical meaning or application (within that physics) unless the laws of motion hold. Therefore, "we know how to give empirical meaning and application to the law of universal gravitation," clarifies Friedman, solely "by presupposing that the laws of motion are true" (2001: 75). These laws work as *a priori* principles. So, if they are untrue, there cannot exist the frame of reference in which they hold, entailing that "the question of the empirical truth (or falsity) of the law of universal gravitation," states Friedman, "cannot even arise" (Ibid).

More precisely, universal gravitation entails absolute acceleration that implies motion. Natural deduction could display their relationship, for instance, by applying the modus ponens as follows[10].

(1) Law of Gravitation $\rightarrow$ *(2)* Laws of Motion

$F = G\frac{m_1 m_2}{r^2}$

First Law (Inertia): $F_{net} = 0$ if velocity is constant
Second Law (Acceleration): $F_{net} = ma$
Third Law (Action & Reaction): $F_{AB} = -F_{BA}$

Notwithstanding, logical empiricists disagree on the notion of relativized *a priori*, for some of them (like Schlick) never endorse it. Still, they all present variations of the Kantian *a priori*. Among others, Parrini focuses on the two most discussed instances, namely Schlick's *implicit definitions* (1918) and Reichenbach's *coordinative definitions* (1920), which I address below.

7. Implicit Definitions

Roughly put, Schlick replaces the Kantian *a priori*, based on sensible intuition, with the notion of *implicit definition*, primarily justified by mathematical developments (see Popper 1959: §17). Here no longer intuitions substitute terms such as point or straight line, indefinable with simple concepts. So, Schlick concludes that modern geometrical axioms avoid visual thinking and rely on conventional definitions stating analytic relations.

[10] Below, *(1)* states the law of universal gravitation, namely all objects attract each other with a force (F) that is proportional to the masses of two objects (m_1 and m_2) and inversely proportional to the square of the distance (r) that separates their centers. Finally, G stands for the gravitational constant. In *(2)*, the *first law* states that an object at rest or in motion remains unchanged unless acted upon by another force. Its related equation is $F = dp/dt$ or $F = d(mv)/dt$, where we have the particle's momentum (p), time (t), velocity (v), mass (m), and the differential operator (d); so, the expression 'dp/dt' denotes the derivative of p with respect to t. In this first law of inertia, $dp/dt = 0$ since no force acts on p. The *second law* establishes that an object's net force (F_{net}) is equal to its mass (m) times its acceleration (a). Here, $dp/dt \neq 0$. The *third law* says that when two objects interact, they apply forces to each other of equal magnitude and opposite direction. As an external agent applies a force F to p, this latter changes according to $dp/dt = F$. So, p must apply an equal and opposite force $-F$ to the external agent. The external agent's momentum p (i.e., the product of a particle's mass and velocity) changes according to $dp/dt = -F$.

> In particular, there would no longer be any *tacit* recourse in geometrical proofs to properties whose presence could be established only by observing the figure. Instead, the existence of these properties would have to be deduced in a purely logical manner from the assumptions and axioms, or if that turned out to be impossible, specifically stated in new axioms. (Schlick 1974: 32)

So, ultimate principles (such as geometrical axioms) underlying all proofs, but themselves improvable, don't owe their validity to the suspicious reliability of intuitions. Schlick refers to Hilbert (1902), who introduces basic or primitive concepts (usually indefinable) "in such a fashion that the validity of the axioms that treat of these concepts is strictly guaranteed" (Schlick 1974: 33). Hilbert's solution is "simply to stipulate that the basic or primitive concepts are to be defined just by the fact that they satisfy the axioms" (Ibid). Schlick calls it as "definition by axioms, or definition by postulates, or implicit definition" (Ibid).

Hilbert's method consists of creating "concepts as clearly determined signs" (Schlick 1974: 33) through which we can confidently advance knowledge and draw inferences. So, he grounds deduction on inferences proceeding from statements or judgments alone. For mathematical propositions' validity (and interconnection), words oughtn't to refer to familiar intuitive figures everyone thinks of when hearing them.

> What matters is only that the word means something for which a particular set of statements (the axioms) holds. And exactly the same thing is true of the remaining concepts that occur in these axioms. They too are defined solely by the fact that they stand in certain relations to the other concepts. (Schlick 1974: 34)

Accordingly, Hilbert's geometry begins with a system of propositions whose terms (e.g., point, straight line, plane, between, plane, and the like) have no content or meaning. They instead "acquire meaning only by virtue of the axiom system and possess only the content that it bestows upon them," says Schlick (1974: 34). Indeed, "they stand for entities whose whole being is to be bearers of the relations laid down by the system" (Ibid). Concepts aren't real things at all even if we are somehow obliged to think of that bearer of relations as endowed with some nature of its own. E.g., in the sentence "the point C lies between points A and B on the straight line a" (Ibid), the words 'between' and 'lie upon' mean certain relations among the objects A, B, and C, but don't signify the objective relations we are familiar with. Their meaning is strictly contextual or relative. Indeed, Hilbert's geometry studies mutual relations of concepts independently of their intuitive meanings.

E.g., imagine spherical surfaces passing through a particular point in space and this point afterward removed from space. Then, in the theorems of Euclidean geometry, "wherever the word 'plane' occurs let it signify one of the spherical surfaces," says Schlick, "let the word 'point' signify a point and the words 'straight line' a great circle on a spherical surface, reinterpret the word 'parallel' in an analogous manner, and so forth" (1974: 35). So, we "obtain a set of propositions all of which hold for the system of spheres," (Ibid); however, we also understand by a straight line of, for instance, a Riemannian manifold a great circle of Euclid's plan. So, "our intuitive picture is," claims Schlick, "entirely different in the two cases" (Ibid). Nevertheless, Riemannian structures, although different in intuitive appearance from the straight lines and planes of ordinary geometry, "stand in the same relations to one another and obey the same axioms" (Ibid).

For Schlick, mutual relations of concepts have always been disengaged from their intuitive content. E.g., he points to the Aristotelian modes of inference.

> When we infer "All *S* are *P*" from the two premises "All *M* are *P*" and "All *S* are *M*", the logical relationship holds quite independently of what the symbols 'S', 'M' and 'P' may mean. All that matters is that the concepts stand to one another in the relations specified in the premises. The symbol 'S' can equally well designate men, or ship's propellers, or logarithms. It is thus easy to see that the introduction of any *ambiguous* symbol initiates a separation of content from the purely logical form, a separation which, pursued consistently, leads eventually to the determination of concepts by means of implicit definitions. (Schlick 1974: 36)

Therefore, a deductive scientific theory construction ignores intuitive pictures of basic concepts. It instead exclusively considers what implicit definitions stipulate, i.e., the reciprocal relations of the primitive concepts as expressed in the axioms. So, against Kant, Schlick holds that the ordinary or concrete definitions' "defining process terminates when the ultimate indefinable concepts are in some way exhibited in intuition" (1974: 37). This latter points to something real involving an individual existence. By default, concrete definitions set up the connection between concepts and reality, while implicit definitions reject any association with this latter. So, the network of concepts is above intuitive or real objects. Indeed, it deals with the abstract, which we apply to the intuitive.

Accordingly, Kant errs in assuming the existence of an apodictic knowledge of reality, relying on Euclidean geometry. Paradoxically, Schlick

grants that Kantian intuitions could explain such apodicticity if it were obtainable. So, only an implicit definition "enables us to determine concepts completely and thus to attain strict precision in thinking" (Schlick 1974: 38). For this purpose, however, concept and intuition, thought and reality, must be radically separated; the two spheres relate to one another but cannot join.

8. Coordinative Principles

As Parrini and Friedman hold, the relativized *a priori* bridges the gap "between abstract mathematical structures and concrete physical phenomena" (Friedman 2001: 78). So, this issue becomes central to the logical empiricist agenda. To solve it, Schlick and Reichenbach identify "a special class of non-empirical physical principles," claims Friedman, "variously called coordinating or constitutive principles by Reichenbach, conventions in the sense of Henri Poincaré by Schlick" (2001: 79). Such a coordination[11] has been anticipated by Kant's constitutively *a priori* principles functioning to mediate between algebraic abstractions and applied physics.

Reichenbach revises the Kantian meaning of *a priori* by theorizing the physical notion of cognition. He points to modern physics, where mathematical equations represent all processes. Still, the two sciences importantly differ. Indeed, "the truth of mathematical propositions depends upon internal relations among their terms;" in contrast, "the truth of physical propositions," argues Reichenbach, "depends on relations to something external, on a connection with experience" (1965: 34). Consequently, we ascribe absolute certainty to *(1)* the former kind of assertions and probability to *(2)* the latter. But what about *(3)* their relationship? Let's see *(1-3)* in detail.

(1) Reichenbach defends a quasi-structuralism in mathematics, where any entity is determined by primitive definitions (i.e., axioms) whose terms rely on the other defining terms belonging to a shared framework.

> The *mathematical object* of knowledge is uniquely determined by the axioms and definitions of mathematics. The definitions indicate how a term is related to

[11] Measurements help determining the values of mathematical functions. without it, any theory would remain pure, and not empirical theory, "if its terms were not linked to measurement procedures" (Van Fraassen 2008: 115). This linkage raises the problem of coordination; namely, "*How can an abstract entity, such as a mathematical space, represent something that is not abstract*, something in nature?" (Van Fraassen 2006: 537).

> previously defined terms. The mathematical object receives meaning and content within this framework of definitions through an analysis of its differences from and equivalences to other mathematical objects. (Reichenbach 1965: 35).

Accordingly, the axioms present the mathematical rules for defining concepts. All concepts, including the fundamental ones occurring in the axioms themselves, are defined through relations. Like Schlick, Reichenbach justifies his argument by referring to Hilbert. Consider, for instance, his axiom of order II-3 stating that "Of any three points situated on a straight line, there is always one and only one which lies between the other two" (Hilbert 1902: 4). Here, Hilbert describes the properties of 'point,' 'straight line,' and 'between' through a *non-exhaustive* definition, made complete solely by the totality of the axioms. All the entities involved (i.e., 'point,' 'straight line,' and 'between') have the axiom-stated properties, owing their nature to mutual relations that can change. E.g., in projective geometry, 'straight line' and 'point' are interchanged, preserving the truth of related theorems since "their axiomatically defined relations are symmetrical for the two concepts" (Reichenbach 1965: 35), although, as Schlick noticed (1974), our intuition depicts the two concepts dissimilarly, ascribing different contents to the axioms.

So, mathematical definitions exhibit a peculiar mutuality, "in which one concept always defines another without the need of referring to 'absolute definitions'" (Reichenbach 1965: 35-6). Mathematical assertions are, therefore, absolutely certain as "they merely represent new combinations of known concepts according to known rules" (1965: 36).

(2) Reichenbach argues that the method of representing physical events relies on mathematical equations. It defines one magnitude in terms of others by relating them to increasingly general magnitudes, up to the axioms. "Yet what is obtained," he claims, "is just a system of mathematical relations," which lacks a statement of its significance, namely "the assertion that the system of equations is *true for reality*" (1965: 36).

> The *physical object* cannot be determined by axioms and definitions. It is a thing of the real world, not an object of the logical world of mathematics. (Reichenbach 1965: 36)

(3) Therefore, the internal coherence of mathematics doesn't suffice for physical truths. These latter also entail a precise relation between equations and physical phenomena.

> The physical relation can be conceived as a coordination: physical things are

> coordinated to equations. Not only the totality of real things is coordinated to the total system of equations, but *individual* things are coordinated to *individual* equations. The real must always be regarded as given by some perception. (Reichenbach 1965: 36-7)

E.g., to name the earth a sphere, we must coordinate the geometrical spherical figure to a specific visual perception, i.e., a perceptual image of the earth, according to some primitive principles of coordination[12]. For this purpose, Reichenbach refers Boyle's gas law, where we coordinate the formula $p \cdot V = R \cdot T$[13] to direct (e.g., feelings) and indirect (the position of a monometer's pointer) perceptions of gas. Indeed, "our sense organs mediate between concepts and reality" (Reichenbach 1965: 37).

Further, such coordination has a peculiar nature. E.g., we can establish a correspondence between two sets by coordinating every element of one set with an element of the other. For this purpose, "the elements of each set must be defined," argues Reichenbach, i.e., "for each element there must exist another definition in addition to that which determines the coordination to the other set" (1965: 37). Yet, although the 'equations' (i.e., the conceptual side of the coordination) are uniquely defined, the 'real' (i.e., the side dealing with the cognition of reality) isn't. Reichenbach overcomes this issue as follows.

> The definition results from a coordination of things to equations. Thus, we are faced with the strange fact that in the realm of cognition two sets are coordinated, one of which not only attains its order through this coordination, but whose elements are *defined by means of this coordination*. (Reichenbach 1965: 40)

Therefore, coordinating principles define reality according to our perceptions and mathematical equations. In this regard, they resemble Kant's conceptualiza-

[12] Within the context of any particular scientific theory, Reichenbach distinguishes between two types of cognitive principles: *(a)* the *axioms of connection* defining empirical laws that involve already well-defined terms and concepts; and *(b)* the *axioms of coordination*, namely non-empirical principles laid down antecedently to ensure that empirical well-definedness.

[13] Today's formula for the ideal gas is $PV = nRT$. In this equation, P stands for pressure and V for volume, n denotes the total amount of gas (measured in terms of moles); finally, R is the universal gas constant, and T the temperature. Let's briefly recall that a **mole** is the measuring unit for *amount of substance* and contains exactly $6.02214076 \cdot 10^{23}$ elementary particles, such as atoms, molecules, ions, and others.

tion of our sensible intuitions employing rules, namely schemata and pure principles of understanding.

Like Reichenbach's, Kant's actual object of cognition derives from the employment of formal structures on blind modifications of sensibility, namely raw sensations. Accordingly, the matter of the cognitive object represents a yet-to-be-determined empirical condition, *a posteriori*. Kant conceives of such determination in two steps, mainly focusing on its possibility or formality. *(1)* Categories unify the manifold intrinsic to our inner sense, shaping the corresponding *schemata*. Indeed, for the content of a category per se is initially derived from the logical structure of judgments alone, "it must be made applicable to objects whose form has thus far been specified solely by the pure forms of space and time," say Guyer and Wood (1998: 10). So, schemata associate categories to a form or relation in intuition, particularly an inner temporal one. *(2)* The *principles of pure understanding* define the rules for applying these schemata to empirical judgments based on our spatial outer sense. Accordingly, "the *use* of [those] schemata in turn depends upon judgments about the *spatial* properties and relations of at least some objects of empirical judgment" (Ibid).

> The *analytic of principles* will accordingly be solely a canon for the *power of judgment* that teaches it to apply to appearances the concepts of the understanding, which contain the condition for rules *a priori*. (Kant 1787: A132/B171)

"Just like Kant's synthetic *a priori* principles, principles of coordination assign conceptual structures to the realm of experience," says De Boer, and "bridge the gap between the conceptual and the sensible" (2010: 517). For Reichenbach, "they ultimately define real objects and real events;" therefore, "we may call them constitutive principles of experience" (1965: 49). So, he refers to Kant's schemata.

Unlike Kant, Reichenbach holds that "the content of every perception is far too complex to serve as an element of coordination" (1965: 40). Before coordination, we must sort out relevant from irrelevant aspects of our perception; namely, we must establish order among them. However, "such a coordination presupposes the equations, or the laws expressed in them" (Ibid). So, Reichenbach maintains that physical knowledge relies entirely on coordination. Indeed, "only a cognitive judgment," he argues, that is an act of coordination, can decide whether the sensation of a tree corresponds to a real tree" (1965: 41), and not to a hallucination.

So, perceptions don't define what is real. Therefore, the elements of the universal set remain undefined since "one side of the cognitive process contains an undefined class" (Reichenbach 1965: 42)[14].

> Thus, it happens that individual things and their order will be defined by physical laws. The coordination itself creates one of the sequences of elements to be coordinated. (Reichenbach 1965: 42)

Nevertheless, the two sides of knowledge maintain a mutual relationship. Indeed, "the defined side does not carry its justification within itself," argues Reichenbach, as "its structure is determined from outside" (1965: 42). So, the coordination to undefined elements is restricted by experience, not arbitrary. Therefore, knowledge preserves an empirical determination or character.

> We notice the strange fact that it is the defined side that determines the individual things of the undefined side, and that, vice versa, it is the undefined side that prescribes the order of the defined side. *The existence of reality is expressed in this mutuality of coordination.* (Reichenbach 1965: 42)

Therefore, this mutuality attests to what is real. It also guarantees truth, which consistently derives from correct coordination, correlating to experience data. As Reichenbach states, "contradictions are discovered by observation" (1965: 43). To be true, a theory must continuously lead to consistent coordination. Like Schlick[15], Reichenbach consistently defines "*truth in terms of unique coordination*" (Ibid). Therefore, perceptions play a crucial role in the cognitive process since they "*furnish the criterion for the uniqueness of the coordination*," he claims (1965: 44).

Despite the differences, Reichenbach underlines the continuity with Kant. His theory of cognitive coordination straightforwardly answers Kant's question, "How is pure natural science possible?" (B20). In a Kantian fashion, 'possible' has a logical, not a psycho-physical meaning; "it pertains to the logical

[14] For Reichenbach, the coordination occurs between a given set of mathematical equations and a completely undetermined reality. Only assigning these equations to experience turns this latter into a proper domain of physics. Indeed, coordinating principles define the individual elements of reality and in this sense constitute the real object. Therefore, like Kant, Reichenbach holds that physics relies on rules unifying pure thought (exemplified by mathematics) and sensible experience.

[15] About the correspondence of judgments with facts, Schlick states that, "a judgment that *uniquely designates* a set of facts is called *true*" (1974: 60).

conditions of a coordination" (Reichenbach 1965: 47). Hence, Reichenbach rephrases Kant's question as follows, "*By means of which principles will a coordination of equations to physical reality become unique?*" (1965: 48). Accordingly, these epistemological principles of coordination "are equivalent to Kant's synthetic a priori judgments" (Ibid).

Therefore, Parrini's weak rejection of the Kantian *a priori* finds evidence in the early work of Reichenbach. As De Boer argues, he "aimed to transform rather than abolish Kant's notion of synthetic *a priori* principles" (2010: 508); for her, the differences with Kant have been overestimated by logical empiricists and their readers such as Friedman. These claims need clarification.

Friedman weds Kant's transcendentalism to Newton's mechanics and Euclid's geometry. If correct, Einstein's new physics demands a detachment from the Kantian *a priori*, stemming from an obsolete paradigm. Yet De Boer separates Kant's synthetic *a priori* principles from Euclidian geometry and Newtonian physics, which instantiate but don't demonstrate the validity of those principles. Instead, she thinks such validity has a metaphysical nature, traceable back to Leibniz and Hume. Whereas Friedman believes that Kant abandoned classic metaphysics (see 1992: 37-8), De Boer contends that he seeks to reconcile metaphysics and Newton's scientific paradigm. However, these two don't overlap since the first can ground any science without restriction.

De Boar convincingly argues that the conditions constructing *(a)* an object of cognition and those building *(b)* a physical law differ. It's always the case that *(a)* entails *(b)*, but the opposite doesn't hold.

> [...] the synthetic *a priori* principles treated in the *Critique* merely constitute necessary rules for determining the spatio-temporal, law-governed relations between given representations - whatever the actual content of these relations may be. (De Boer 2010: 510)

"Unlike laws of physics [*b*], the principles of pure understanding do not depict the world, but constitute the 'rules of the pure thinking of an object' [*a*]" (De Boer 2010: 510). These principles offer "perspectives that we must necessarily adopt," says de Boer, "to turn phenomena into objects of knowledge" (Ibid). "Without such synthetic *a priori* root-principles it would not be possible for us to establish laws of physics proper" (Ibid). So, De Boer sharply divides *(a)* and *(b)*. Accordingly, "the synthetic *a priori* principles of pure understanding" represent "the root-principles by dint of which something can become an object of

knowledge in the first place," namely, they delimit the domain "within something can be treated as an object" (Ibid)[16].

These claims justify the central tenet of Parrini's notion of 'weak rejection,' namely the Kantian *a priori* must be retained as it grounds every cognitive object before being employed in a scientific theory. However, is it even possible? Isn't the other way around, namely that a scientific theory shapes its cognitive objects from the outset? If so, Parrini's 'strong rejection' of Kant's *a priori* would offer a more viable reading of the logical-empiricist epistemology.

Still, textual evidence supports De Boer's analysis. For Kant, laws of nature are principles of the empirical use of the understanding. The necessity they carry derives *a priori*, namely it's valid prior to all experience. "But," clarifies Kant, "without exception all laws of nature [*b*] stand under higher principles of the understanding [*a*], as they only apply the latter to particular cases of appearance" (A159/B198). For De Boer, showing how *(b)* depends on *(a)* falls outside the scope of the first *Critique* and pertains to the later works (see Kant 1796).

Notwithstanding, Kant claims that not only *(b)* necessarily requires *(a)*, but also that *(b)* represents the true purpose of cognition. Therefore, when Friedman privileges the route from the principles of pure understanding to Newtonian physics laws, he retraces Kant's journey (see Kant 1796). So, do *the analogies of experience* become meaningful only through their instantiation (or realization) in the Newtonian laws of motion? Generally, does Kant's *a priori* depend on Newton's physics? Friedman and De Boer would reply yes and no, respectively. Although Kant's texts refer to Euclid and Newton (Friedman's correctly builds his reading on this reference), his *a priori* can nevertheless be detached from their geometry and physics (as De Boer suggests).

Although De Boer stresses the similitude with the Kantian *a priori*, logical empiricists reject some of its properties, especially the independence of all experience and the apodicticity. As confuted by sciences, absolute necessity

[16] But how *(a)*'s grounding of *(b)* work? De Boer explains it as follows. "Thus, far from telling us something about the world, the principle based on the pure concept of quantity merely states the rule that every intuited object has an extension and, hence, can be determined mathematically. According to Kant, it is only on the basis of this principle that physics can apply pure mathematics to objects of experience (A165/B206). The category of substance, for its part, yields the rule that scientific knowledge must necessarily distinguish between that which changes over time and that which constitutes the self-identical substrate of such changes. Otherwise, scientists would neither be able to determine something as an object, nor to determine the relation between objects." (De Boer 2010: 512).

looks like an undisputable flaw. Reichenbach's early conception of the relative *a priori* is no exception. Therefore, the idea that constitutive scientific principles can be *a priori* and revisable perfectly instantiates Parrini's notion of 'weak rejection,' balancing continuity and discontinuity with Kant. Reichenbach describes this balance as follows.

> The *concept of the a priori* is fundamentally changed by our investigations. Because of the rejection of Kant's analysis of reason, one of its meanings, namely, that the a priori statement is to be eternally true, independently of experience, can no longer be maintained. The more important does its second meaning become: that the a priori principles constitute the world of experience. (Reichenbach 1965: 77)

Hence, revisability and sensible dependency define Reichenbach's 'weak rejection' in 1920. Later (1924, 1928), he softens his initial criticism of Schlick's 'strong rejection' of Kant's *a priori*, based on the conventionalist characterization of this latter and the denial of its constitutive function.

9. Final Remarks

Parrini demonstrates that leading neo-positivists, although designing dissimilar theories of scientific knowledge, recognize the relevance of the Kantian *a priori*. The refusal of its distinctive features of apodicticity and (sometimes) constitutive function never leads to "a naïve empiricist position that neglects the problems of conceptualization and believe it can treat all scientific statements in the same way, observing that [after all] *everything is experience*" (Parrini 2002: 150).

Supporting evidence first comes from *(a)* Reichenbach's constitutive *a priori* that maintains the relevant aspects of Einstein's relativist construction: abstract-theoretical and distinctly empirical. His coordinative principles[17] connect cognitively dependent but metaphysically separate levels of conceptualization, the theoretical and the empirical. Consistently, Schlick defines scientific theories as "empirically interpreted deductive-axiomatic systems, whose primitive no-

[17] For Reichenbach (1920), the rational components of knowledge (i.e., the coordinative definitions) rely on experience. As Parrini points out, such experience-dependency is conceived holistically. Coordinative principles don't say what is individually known but how the whole experience is knowable. In this sense alone, they are a priori. Namely, *a priori* means before knowledge, but not for all time and not even independent of experience.

tions acquire empirical value by using a particular type of associative links (largely conventional in Poincaré's fashion)" (Parrini 2002: 150-1). His definition bars any theory of truth relying on correspondence or congruence.

(b) Carnap (1937) recovers Reichenbach's notion of relativized *a priori*. His analysis of syntactically specified linguistic frameworks reveals arbitrary forms of language systems embodying rules of any kind of logic. So, no correctness tests the pragmatic choice of this latter, subject to a *principle of tolerance* alone (Carnap 1937: §17). These frameworks can be other than purely mathematical and include physical laws. In this case, they are physical languages. Here, "in addition to purely mathematical principles, we also formulate physical hypotheses and test them via the logical-mathematical deduction of protocol-sentences" (Friedman 1999: 68) – see Carnap 1932.
Within any physical language, Carnap separates logical from physical principles, L-rules (i.e., analytic sentences) from P-rules (i.e., synthetic sentences), all revisable and subject to pragmatic and Duhemian holistic considerations (see Duhem 1906). However, their revisions differ. Changing the L-rules affects a language and the meanings of its terms, whereas changing the P-rules only modifies synthetic or empirical sentences formulated within a given (and therefore fixed) language. Consistently, "Carnap's L-rules or analytic sentences," says Friedman, "can be profitably viewed as a precise explication of Reichenbach's notion of the constitutive or relativized *a priori*" (1999: 69)[18], which straightforwardly supports Parrini's *weak rejection*.

(c) Einstein's relativistic physics and Kant's synthetic *a priori* aren't simplistically incompatible. Though revisable and contingent (i.e., experience-dependent), the *a priori* remains constitutive (although Schlick doesn't distinguish the two features[19]). Indeed, as Reichenbach argues, "empirical observation can bring out *implicit contradictions* in systems of principles devoid of *explicit contradictions*" (Parrini 2002: 154). The self-evident principles of reason may generate contradictions, unidentifiable within a system, but evident when this system is applied to experience. Still, for Reichenbach, Einstein's special relativi-

[18] Also, "Carnap's account, like Reichenbach's, thus yields a theory specific (language specific) distinction between two intrinsically different types of principles and therefore a true relativized *a priori*" (Friedman 1999: 70).
[19] For Schlick (1921), from the empirical standpoint, the allegedly constitutive principles of knowledge look like general hypotheses or conventions. As hypotheses, these principles aren't *a priori* (lacking apodicticity); as conventions, they aren't synthetic either.

ty theory proves that no object of cognition is possible without coordinative principles determining it.

> The theory of relativity certainly sanctioned the end of the transcendental method and its claim to limit experience by guaranteeing the universal validity of certain presuppositions of knowledge. Still, it also substantially confirmed the Kantian and neo-Kantian conception according to which 'the object of knowledge, the thing of appearance [says Reichenbach] is not immediately given' because it's instead a structure of reference built through categories and based on intuition.' (Parrini 2002: 158)

(d) However, even as the later Reichenbach (1924, 1928), adjusting to Schlick's criticism, drops the apriority's constitutive feature, he never falls into a sensistic empiricism neglecting the issue of conceptualization.

> Finally, Reichenbach's recovery of the empirical character and the objective value of scientific statements (such as the geometric descriptions of physical space) goes through a careful reconsideration of the Kantian aspects characterizing his starting position, carried out according to Poincaré's conventionalism and Schlick's theory of knowledge. A fundamental element of this conception becomes the new interpretation of the decisional components of the theories. They are no longer understood as in the early 20s in terms of constitutive principles of the concept of an object but conceived as conventional stipulations – although not arbitrary [...] – having a linguistic-definitional nature devoid of empirical content and genuine cognitive value. (Parrini 2002: 162)

Hence, Parrini identifies a *weak rejection* of the Kantian *a priori* with the core of logical empiricism. Still, he never minimizes the detachment from Kant. In 1935, Reichenbach confirmed Parrini's view by stating that modern science no longer believes in the legislative capacity of pure reason. In contrast to Kant, everything we know about the world derives from experience, and the data's transformations are purely tautological and analytic. His synthetic *a priori* laws are actually empirical (e.g., laws of space and time, causality, conservation of substance, and others). Similarly, mathematical formations aren't synthetic *a priori* but are analytic alone. So, "to demonstrate all this," says Reichenbach, "empiricism joined logic, and such logical empiricism defines the contemporary form of that historical stream of philosophy born from the logical critique of science" (1935: 33).

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REFERENCES

Abela, Paul (2002), *Kant's Empirical Realism* New York: Oxford University Press.

Allais, Lucy (2015), *Manifest Reality. Kant's Idealism and his Realism*. New York: Oxford University Press.

Allison, Henry E. (2004), *Kant's Transcendental Idealism*. New Haven and London: Yale University Press.

Ayer, Alfred Jules (1952), *Language, Truth and Logic*. New York: Dover.

Banks, Erik C. (2014), *The Realistic Empiricism of Mach, James, and Russell. Neutral Monism Reconceived*. Cambridge: Cambridge University Press.

Beth, Evert W. (1956-57) "Über Lockes 'allgemeines Dreieck'." *Kant-Studien* 48, 361-80.

Carnap, Rudolf (1930), "Die alte und neue Logik." *Erkenntnis* 1, 12-26.

Carnap, Rudolf (1931), "*Überwindung* der *Metaphysik* durch logische Analyse der Sprache." *Erkenntnis* 2, 219-41.

Carnap, Rudolf (1932), "Über Protokollsatze." *Erkenntnis* 3, 215-28.

Carnap, Rudolf (1937 [1934]), *The Logical Syntax of Language*. A. Smeaton (transl.). London: Kegan Paul. Originally published as *Logische Syntax der Sprache*. Wien: Springer.

Carnap, Rudolf (2005 [1928]). *The Logical Structure of the World*. Chicago and La Salle: Open Court.

Da Silva, Jairo José (2017), "On Color: The Husserlian Material a Priori." In Silva Marcos (ed), *How Colours Matter to Philosophy*, Springer 97-105.

De Boer, Karin (2010), "Kant, Reichenbach, and the Fate of A Priori Principles." *European Journal of Philosophy* 19/4, 507-31.

Duhem, Pierre ([1906] 1954), *The Aim and Structure of Physical Theory*. Princeton: Princeton University Press. Originally published as *La Théorie Physique. Son Objet, sa Structure*. Paris: Chevaliere & Rivière, 1906.

Einstein, Albert (1954 [1919]), "What is the Theory of Relativity?" In *Ideas and Opinions*. New York: Crown Publishers, pp. 227-32.

Einstein, Albert (1954 [1921]), "Geometry and Experience." In *Ideas and Opinions*. New York: Crown Publishers, pp. 232-46.

Euclid (1908, [c. 300 BC]), *The Thirteen Books of the Elements*, Vol.1 (Books I and II). Sir Thomas L. Heath (ed). Cambridge: Cambridge University Press.

Frank, Philipp (1949), *Modern Science and Its Philosophy*. Cambridge MA: Harvard University Press.

Friedman, Michael (1990), "Kant on Concepts and Intuitions in the Mathematical Sciences". *Synthese* 84, 213-57.

Friedman, Michael (1992), *Kant and the Exact Sciences*. Cambridge, MA: Harvard University Press.

Friedman, Michael (1999), *Reconsidering Logical Positivism*. New York: Cambridge University Press.

Friedman, Michael (2001), *Dynamics of Reason*. Stanford, CA: CSLI Publications.

Friedman, Michael (2007), "Coordination, Constitution, and Convention: The Evolution of the A Priori in Logical Empiricism." In A. Richardson and Uebel (eds), *The Cambridge Companion to Logical Empiricism*. New York: Cambridge University Press, 91-116.

Friedman, Michael (2009), "Einstein, Kant, and the Relativized A Priori." In M. Bitbol, P. Kerszberg, J. Petitot (eds.), Constituting Objectivity. Springer Science, 35-50.

Guyer, Paul (1987), *Kant and the Claims of Knowledge*. New York: Cambridge University Press.

Guyer, Paul and Woos, Allen (1998), *Introduction to the Critique of Pure Reason*. In Immanuel Kant, *Critique of Pure Reason*. P. Guyer/A.W. Wood (eds). Cambridge: Cambridge University Press, 1-80.

Hanna, Robert (2022), "Kant's Theory of Judgment", *The Stanford Encyclopedia of Philosophy*, Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/spr2022/entries/kant-judgment/>.

Harper, William (1986), "Kant on the A Priori and Material Necessity." In Robert E. Butts (ed), *Kant's Philosophy of Physical Science*. Dordrecht: Kluwer, pp. 239-73.

Harvey, Matthew (2015). *Geometry Illuminated. An Illustrated Introduction to Euclidean and Hyperbolic Plane Geometry*. Washington, DC: The Mathematical Association of America.

Hatfield, Gary (2002), *Translator's Introduction* to *Prolegomena*. In H. Allison and P. Heath (eds), *Theoretical Philosophy after 1781*. New York: Cambridge University Press, 29-47.

Hertz, Heinrich (1894), *Die Prinzipien der Mechanik in neuen Zusammenhang dargestellt*. In *Gesammelte Werke*, Volume III. Leibzig: Barth.

Hilbert, David (1902 [1899]) *The Foundations of Geometry*. E. J. Townsend (transl.). Chicago: Open Court 1902. First published in German as *Grundlagen der Geometrie*. Leipzig: Teubner, 1899.

Hintikka, Jaakko (1967), "Kant on the Mathematical Method." *The Monist*, Vol 51, 352-75. Republished in C. J. Posy (ed), *Kant's Philosophy of Mathematics*. Dordrecht: Kluwer, 1992: 21-42.

Husserl, Edmund (2001 [1900/1901]), *Logical Investigations*, Vol. II. Dermot Moran (ed), J.N. Findlay (transl.). London & New York: Routledge. First published as *Logische Untersuchungen*. Halle: Niemeyer, 1900/1901.

Jech, Thomas (2003 [1997]), *Set Theory*. Berlin-Heidelberg: Springer.

Kant, Immanuel (1992). *Lectures on Logic*. J. M. Young (ed). New York: Cambridge University Press.

Kant, Immanuel (1998 [1787]), *Critique of Pure Reason*. P. Guyer/A.W. Wood (eds). Cambridge: Cambridge University Press.

Kant, Immanuel (2002 [1783]). *Prolegomena to Any Future Metaphysics That Will Be Able to Come Forward as Science*. In H. Allison and P. Heath (eds), *Theoretical Philosophy after 1781*. New York: Cambridge University Press, 29-170.

Kant, Immanuel (2002 [1786]). *Metaphysical Foundations of Natural Science*. In *Theoretical Philosophy after 1781*. In H. Allison and P. Heath (eds), *Theoretical Philosophy after 1781*. New York: Cambridge University Press, 171-270.

Lanfredini, Roberta (ed) (2006), *A Priori Materiale. Uno Studio Fenomenologico*. Guerini e Associati.

Langton, Rae (1998), *Kantian Humility. Our Ignorance of Things in Themselves*. New York: Oxford University Press.

Mach, Ernst (1897), *The Analysis of Sensations and the Relation of the Physical to the Psychical*. Chicago: Open Court.

Neurath, Otto (1973 [1929]), "Wissenschaftliche Weltauffassung: Der Wiener Kreis." In Empiricism and Sociology. Dordrecht: D. Reidel Publishing Company, pp. 299-318.

Newton, Isaac (1999 [1687]), *The Principia. Mathematical Principles of Natural Philosophy*. B. Cohen and A. Whitman (eds). Berkeley: University of California Press.

Oliva, Luca (2018), "On Kantian Intuitions." In P. Muchnik and O. Thorndike (eds), *Rethinking Kant*, Vol. 5. North America Kant Society Series. Newcastle upon Tyne: Cambridge Scholars Publishing, 71-92.

Parsons, Charles (1969), "Kant's Philosophy of Arithmetic." In Sidney Morgenbesser, Patrick Suppes, and Morton White (eds), Philosophy, Science and Method: Essays in Honor of Ernest Nagel. New York: St. Martin's Press. Republished in C. J. Posy (ed), Kant's Philosophy of Mathematics. Dordrecht: Kluwer, 1992: 135-58.

Parrini, Paolo (1994) "On Kant's Theory of Knowledge: Truth, Form, Matter." In P. Parrini (ed) *Kant and Contemporary Epistemology*. Dordrecht: Kluwer, 195-230.

Parrini, Paolo (1998), *Knowledge and Reality. An Essay in Positive Philosophy*. Springer. Modified version of Parrini, Paolo (1995), *Conoscenza e Realtà*. Roma-Bari: Laterza.

Parrini, Paolo (2002), *L'Empirismo Logico. Apetti Storici e Prospettive Teoriche*. Roma: Carocci.

Parrini, Paolo (2017), "Introduzione. Mach Scienziato-Filosofo." In Ernst Mach, *Conoscenza ed Errore. Abbozzi per una Psicologia della Ricerca*. Milano: Mimesis.

Peruzzi, Alberto (2006), "The Meaning of Category Theory for 21st Century Philosophy." *Axiomathes* 16, 425-60.

Piazza, Tommaso (2004), "The Quest for the Synthetic A Priori: Husserl and Schlick's Debate Revisited." In Arkadiusz Chrudzimski and Wolfgang Huemer (eds), *Phenomenology and Analysis: Essays on Central European Philosophy*. Frankfurt: Ontos, 233-56.

Poincaré, Henri (1905 [1902]) *Science and Hypotheses*. New York: The Science Press.

Pojman, Paul (2020), "Ernst Mach," *The Stanford Encyclopedia of Philosophy*, Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/win2020/entries/ernst-mach/>.

Popper, Karl (1959) *The Logic of Scientific Discovery*. London and New York: Hutchinson & Co.

Reichenbach, Hans (1936), "L'empirisme logistique et la désagrégation de l'a priori'." *Actes du Congrès international de philosophie scientifique*, Paris 1935, Vol. I: *Philosophie scientifique et empirisme logique*. Paris: Hermann: 28-35.

Reichenbach, Hans (1965 [1920]) *The Theory of Relativity and A Priori Knowledge*. Berkeley and Los Angeles: University of California Press. Originally published as *Relativitätstheorie und Erkenntnis Apriori*. Berlin: Springer, 1920.

Reichenbach, Hans 1969 [1924], *Axiomatization of the Theory of Relativity*. M. Reichenbach (transl.). Los Angeles: University of California Press. Originally published as *Axiomatik der relativistischen Raum-Zeit-Lehre*. Braunschweig: Vieweg, 1924.

Reichenbach, Hans 1958 [1928], *The Philosophy of Space and Time*. M. Reichenbach and J. Freund (transl.). New York: Dover. Originally published as *Philosophie der Raum-Zeit-Lehre*. Berlin: de Gruyter, 1928.

Russell, Bertrand (1897), An Essay on the Foundations of Geometry. Cambridge UK: Cambridge University Press.

Schlick, Moritz (1917), *Raum und Zeit in der gegenwärtigen* Physik. Berlin: Springer.

Schlick, Moritz (1921), "Kritizistische oder empiristiche Deutung der neuen Physik?" *Kant Studien* 26 (1-2), 96-111.

Schlick, Moritz (1948 [1930]) "Is There a Factual *a Priori*?" In H. Feigl and W. S. Sellars (eds), *Readings in Philosophical Analysis*. Ridgeview, 277-85. First published as "Gibt es ein materiales Apriori?" In *Wissenschaftlicher Jahresbericht der Philosophischen Gesellschaft an der Universität zu Wien für das Vereinsjahr 1930/31*.

Schlick, Moritz (1974 [1918]), *General Theory of Knowledge*. New York-Wien: Springer. First published in German as *Allgemeine Erkenntnislehre*. Berlin: Springer 1918.

Simons, Peter (1992), *Philosophy and Logic in Central Europe from Bolzano to Tarski*. Dordrecht: Kluwer.

Strawson, Peter F. (1966), *The Bounds of Sense. An Essay on Kant's Critique of Pure Reason*. New York and London: Routledge.

Van Cleve, James (1999), *Problems from Kant*. New York: Oxford University Press.

Van Fraassen, Bas C. (2006), "Representation: The Problem for Structuralism." *Philosophy of Science* 73, 536-47.

Van Fraassen, Bas C. (2008), *The Problem of Coordination*. Ch. 5 in *Scientific Representation: Paradoxes of Perspective*. New York: Oxford University Press, 115-39.

Westphal, Kenneth R. (2004). *Kant's Transcendental Proof of Realism*. New York: Cambridge University Press.

Westphal, Kenneth R. (2021), *Kant's Critical Epistemology. Why Epistemology Must Consider Judgment First*. New York and London: Routledge.

Wittgenstein, Ludwig (1929), "Some Remarks on Logical Form." In *Aristotelian Society Supplementary Volume*, Vol. 9, Issue 1, 162-71.

Wittgenstein, Ludwig (1961 [1922]), *Tractatus Logico-Philosophicus*. New York: Routledge.