The Problem of Generalities. Carnap and Kaufmann in Comparison (1928-1934)

Felice Masi[*]
felice.masi@unina.it

ABSTRACT

Between 1928 and 1934, Rudolf Carnap and Felix Kaufmann engaged in an intense discussion on the difference between two types of generalities. Kaufmann introduces the topic, but both consider it decisive for the theory of meaning and the philosophy of mathematics. That is two of the central issues in the discussions of the *Vienna Circle*, one related to the confrontation with *Tractatus*, the other to that with *Principia Mathematica.* Through the difference between generalities, Kaufmann brings Husserlian theories of abstraction and concept into Vienna. For the first, and perhaps the only time, a live confrontation between phenomenology and logical neo-empiricism takes place.After a brief exposition of state-of-the-art on generalities in contemporary literature, the paper presents in the first paragraph the Kaufmannian definition of the two generalities and its application to the philosophy of mathematics, in the second, the Husserlian presuppositions of the distinction in the third the first phase of the debate between Kaufmann and Carnap (1928-1931) and in the fourth the second phase (1928-1934). The main result of the confrontation is the definition of thing language and the discussion on the extent of its extension, decisive arguments for the liberalisation of the empirical criterion of significance and thus for the fate of 20th-century empiricism

In contemporary literature, when speaking of generalities or general statements, two distinctions are considered: between a restricted and an unrestricted generality and between a generality that admits an instance-based explanation and one that does not.

For the first distinction, the two positions at stake can be exemplified respectively by Dummett (1973, 1991, 1996) and Quine (1948) or, more recently, by Williamson (2003). According to the former, based on an anti-realist conception of truth and a neo-verificationist theory of meaning, one can

[*] University of Naples Federico II, Naples, Italy..

only admit a generality over a delimited domain whose elements are directly or indirectly constructible to be able to answer the question of whether it is the case that a generality applies to a given instance. The latter, on the other hand, uses a more complex strategy that has at least two levels. Firstly, they do not agree with the paradoxically realistic effect of constructivism on generalities, i.e. the limitation to those extensions that are really satisfiable. This realism on generalities would entail metalinguistic pessimism as well as deviant logic. Secondly, they argue that it is enough to examine the form that the restricted generality takes to realise how a non-restricted generality must be presupposed to obtain it. For let us take 'all men are mortal' and understand it as a general statement restricted to men only; such a restricted general statement should be understood as 'for all x, if x is a man, then x is mortal', i.e.

$$\forall x(Ux \rightarrow Mx)$$

in which universal quantification on *x* needs to be unrestricted so that the predicate U then restricts the scope of M. This is not to deny the context effects on generalisation when its scope needs to be identified. Even allowing for this, however, the only way out that would remain for the generality-relativist would be that of kind-generalisation. In statements such as 'no monkeys talk', 'all donkeys bray' or 'all electrons move at a speed slower than the speed of light', the generality-relativist could only assume that generality is restricted over an arbitrary extension of the domain and that there is no need to assume an unrestricted generality if he had semantic resources at his disposal that for the generality-absolutist are even taboo (Williamson 2003, 443). But that would move us to the metaphysical plane where monkey, donkey and electron are kinds and the terms expressing them sortal.

The moral of the story would be that the restriction of generality is anything but a demonstration of modesty since it implies much more demanding commitments than those of the generality-absolutist who basically admits, out of habit and instrumentally, that the *all* varies on everything, whatever it is and even if he does not know what it is.

The second distinction apparently has a more recent history and dates back to the introduction in Carlson (1977) of generic generality and plural quantifiers, both existential and universal, in Boolos (1984, 1985). A generic generality is that of 'crows are black' and its formalisation would be

$$\text{Gen } x\,[\text{Ravens}(x)]\ [\text{Black}(x)],$$

and not

$$\forall x(Rx \rightarrow Bx).$$

Otherwise, an example of plural statement is ‘there are some apples on the table’ and could be formalised in a Plural-First-Order Language (L_{PFO}) with

$$\exists xx\forall u(u \prec xx \rightarrow Au \wedge Tu),$$

i.e. there are *x*’s and for all *u*’s, if u is one of the *xx*’s, then it is an apple and it is on the table (Linnebo 2022b), instead of with

$$\exists x\forall u(Au \rightarrow Tu \wedge x = u).$$

More interesting is the case in which at least one of the predicates used cannot be understood distributively (i.e. for each of the elements of the domain over which the quantification varies) but only collectively. For if I say that ‘some apples on the table form a circle’, the predicate ‘form a circle’ does not apply to each apple on the table but to the apples taken as a whole. Thus, despite their differences, generic generality and plural generality are united by their interest in non-distributive quantifications and statements concerning arbitrary objects.

The two distinctions between restricted and unrestricted generality and between distributive and non-distributive generality are usually not addressed together. An exception is Linnebo (2022a), who distinguishes the instance-based explication of generality from an explanation “based on general facts about the properties or operations involved in the claim that is generalised”. It should be noted that Linnebo does not consider these two options to be two different ways of understanding quantification but two different ways of explaining it and that the two explanations are not exclusively disjunctive: it is indeed possible to explain a statement such as ‘every whale is a mammal’ both instantially and generically. In a note in which he gives the prehistory of the generic explanation of quantification, Linnebo (2022a) mentions Russell (1908), Weyl (1921), Carnap (1931) and Kaufmann (1931), among others. The latter are the only two essays cited that bear the same year of publication. This apparent coincidence conceals a long story that deserves to be told.

My intention in this essay is to recount this story as a concrete case of a comparison between phenomenology and logical neo-empiricism, a comparison whose relevance has been repeatedly emphasised by Parrini (1980; 1998; 2022), selecting, however, at Preti’s suggestion, the Carnap of inductive logic, the mature Husserl, and the theme of explicitation.

The subject of the essay is the dense exchange, in 1928-1934, between Kaufmann and Carnap on the difference between empirical, i.e. distributive, denumerable, and restricted generality and specific generality. The two terms are defined by Carnap (1928) and the discussion with Kaufmann that followed and by Carnap (1934) and the discussion with Kaufmann that preceded it. The peculiarity, or perhaps I should say the uniqueness, of the confrontation under consideration is that from it arose notions and concepts that were later used by both in the essays they published during those seven years. In other words, although Kaufmann appears as the proponent of that distinction and although Carnap acknowledges this to him by quoting Kaufmann (1931) each time he uses it, if we exclude the references in Kaufmann (1921) to the distinction between generalisation and formalisation and to the difficulties inherent in the Husserlian-Hilbertian conception of definite variety, Kaufmann will only use the notion of specific generality after he has begun the discussion with Carnap.

The essay will be divided into four parts. The first will briefly present Kaufmann's position (1931) and give an initial definition of the difference between generality so that it will be clear from the outset what we are talking about. The second will be devoted to a summary exposition of the perspective with which the early Husserl deals with the topics of generality, abstraction, and natural numbers to prepare a litmus test for pointing out the substantial deviations contained in Kaufmann's analyses. With the third part, we enter into the substance of our story: it will cover the period from 1928 to 1930 and will be devoted to the exchange between Kaufmann and Carnap on Carnap (1928) and, in particular on the distinction between individual and general concepts in § 158, the use of specific generality in Kaufmann (1930, 1931, 1932) and Carnap (1931). The fourth part, which will cover the period from 1931 to 1934, will instead deal with Carnap's and Kaufmann's comparison of the preparatory manuscripts in Carnap (1934), Kaufmann's analyses (1932) and Carnap's comments on them, as well as Carnap (1936)[1] . The conclusions will

[1] In particular, for the first two parts of the essay, in addition to the texts cited and Carnap (2022), the following will be used: 1) for Carnap, the manuscripts RC 028-26-11 (16/12/1928), RC 029-26-10 (5/4/1929), RC 089-64-02 (15-17/11/1929), RC 028-26-07 (23/5/1930), RC 028-26-06 (13/11/1930), RC 028-26-01 (29/11/1932), preserved at the Archives for Scientific Philosophy and consulted at https://valep.vc.univie.ac.at/virtualarchive/public/ASP_Archive/a:373974; 2) for Kaufmann, the manuscript of *Idee*

mention Kaufmann's gradual abandonment of the notion of specific generality from the second half of the 1930s, his not entirely successful attempt to hybridise it with the Deweyian notion of a generic proposition (as distinct from a universal one), and its survival in Carnap (1936-37). The secondary objective of the essay is to shed light on one of the decisive steps that led to Carnap's writing (1934) and the introduction of the thing language, offering documents to reconstruct part of the critical revision that in the Vienna Circle was addressed to the Russellian theory of types, the axiom of reducibility and the Tractarian conception of generality, but also to better understand the context in which the first reception of Gödel (1931) took place.

1.

The difference between generalities and the fact that it had been central in the confrontation with Kaufmann is not often mentioned in the literature on Carnap (with the exceptions of Proust 1986, 456; and to some extent Goldfarb 2009, 115 and Carus 2010, 256). When one even examines the contribution Kaufmann made to the logical and philosophical-mathematical discussions within the Vienna Circle, at best, one concedes, as Nagel (1978, xiv) does, that the admission of a non-extensional generality is little more than a triviality[2] . If one then thinks of the reference found in passing in Popper (1934), one cannot help but be astonished by the simplification of one of the two generalities (the empirical one) into a bad copy of the finite conjunction of the *Tractatus.* Rather, it is precisely in the literature on Wittgenstein, and in particular on the philosophy of mathematics in its intermediate phase, that the distinction be-

der logischen Grundwissenschaft, dated 1923 and included in the *Kaufmanns Nachlass* (KN) held at the *Sozialwissenschaftliches Archiv Konstanz* (SAK) from p. 5220, the Carnap-Kaufmann epistolary (KN p. 8078 to p. 8246), Notebook 15 on *Carnap 'Semantik'* (KN 14137), and the *Logische Prinzipienfragen in der mathematischen Grundlagenforschung* (KN 3204-3254). For the consultation of the *Kaufmanns Nachlass* and permission to publish some of its material here, I would like to thank Jochem Dreher, Director of the SAK.

[2] It should be noted that the example of a specific universal given by Nagel (1978, xi) is incorrect. 'All prime numbers greater than 2 are even' introduces an impermissible universal quantification on ideal singularities. Moreover, the comparison between that example and 'all animals housed at the Milwaukee Zoo weigh more than two kilos' (an example of a restricted individual generality) gives the unspecified impression that the difference between the two generalities corresponds rather to the distinction between a priori and a posteriori. It will be seen that this is not an entirely unfounded impression; but presented in this way, it is undoubtedly circular.

tween generalities is most interesting. In examining Wittgenstein's critical stance on the logicist reduction of mathematics, and on the axiom of complete induction as well as on the problem of impredicativity, Marion (1998, 2008, 45-46) points out that among the solutions available to Wittgenstein at the time was Kaufmann's (1930) distinction between generalities, which was publicised by Carnap (1931). Although the route that would lead Wittgenstein from the definition of universal generalisation as a series of conjunctions to the definition of infinity as a property of a law and not of its extension, adds Marion (1998, 2008, 182) was very similar to that suggested by Kaufmann's finitism, it seems that Wittgenstein disagreed with Kaufmann (1930) as reported by Waismann (1979, 82)[3] .

I have already made it clear that the comparison between Carnap and Kaufmann on the difference between generalities does not start from the definition achieved in Kaufmann (1930) but precedes it and that this definition is also the result of the comparison. Nevertheless, it seems to me appropriate to establish as of now what the difference consists of, emphasising that it has one foot in abstraction theory and one in number theory. This will make it possible to identify its assumptions in Husserl (but also the differences from these) and the articulation of the comparison with Carnap. The main definitions of the difference between generalities are found in Kaufmann (1930, 22-23), with more attention to the dangers of confusion between the two generalities, and Kaufmann (1931, 266), with more attention to abstraction theory and the problem of universals.

In Kaufmann (1930, 135), the order of the fundamental concepts in his philosophy of mathematics is thus determined: "the difference between individual and specific generalities, the elimination of the concept of set in the definition of natural number, the clear understanding of the relationship between cardinal and ordinal number, the result of the analysis of the principle of complete induction and the dissolution of the symbolism of irrational num-

[3] Cf. also Frascolla (1994). Waismann (1979, 84) rather reports on the Wittgensteinian rejection of the question of the priority of ordinals or cardinals and the conception of number as that which remains invariant in enumerations following different orders. It should be added that Waismann was directly involved in the attempt to prove the constructiveness of Kaufmann's (1931) indirect proofs and that Kaufmann (1930, 130) mentions a conversation with Waismann in which he learned that Ramsey was dissatisfied with his solution to the construction of irrational number theory without branching type theory and without AR.

bers". If one, therefore, confuses the two generalities (Kaufmann 1930, 28, 30, 79, 81, 90, 141), the other four concepts are not derivable. The two generalities are defined as empirical and non-empirical (Kaufmann 1930, 15) or individual and specific (Kaufmann 1930, 22-23); the second generality also possesses different levels (Kaufmann 1930, 39). In Kaufmann (1931, 271), the two generalities are also called general and numerical. The basis for this distinction is offered by a theory of abstraction and is therefore justified in epistemological and philosophical-mental terms.

Interestingly enough, in comparison to Kaufmann (1930), Kaufmann (1931) strongly emphasises the interpretation of qualities in terms of tropes and de-idealises invariance[4] : this is one of the visible results of the comparison with Carnap. Whilst, in fact, in Kaufmann (1930, 19) it is sufficient to distinguish between properties of an object and properties of a property, in Kaufmann (1931, 266), it is stated that whenever a certain quality is spoken of, or something is said of the colour green, it is actually meant 'the quality of a certain thing...' and 'things coloured green...'. The semi-conceptualism of Husserlian abstraction theory is thus progressively bent into a seminominalism, i.e. into a *linguistic physicalism* (Dahms 1997): the result will be the construction of a realist phenomenological language restricted to individuals and properties of individuals. Another product of the comparison is the accentuation of a tendency that can already be seen in the transition from Kaufmann (1930) to Kaufmann (1931), namely the increased use of *Verdeutlichung* and the progressive substitution of abstraction. In both Kaufmann (1930, IV) and Kaufmann (1931, 263), it is stated that "the task of philosophy is to clarify concepts through reflection (*Besinnung*)", with a declared reference to *Formal and transcendental logic* (henceforth FTL). Still, in Kaufmann (1931, 273), it is also added that one can only speak of properties (without speaking of the individuals that instantiate them) through clarification.

The difference between the generality of empirical propositions that refer to morphological similarities between individuals (Kaufmann 1931, 266)[5] and the generality of non-empirical propositions that refer to essential traits, to qualities, to the so-being exemplified by several individuals is such that while the individual generality is bound to a specific spatiotemporal

[4] For a recent application of trope theory in a phenomenological context, see Kriegel 2004.
[5] In Kaufmann (1931, 266), morphological similarities or empirical connections also include the laws of nature.

sphere, the specific generality is independent of this sphere, but not entirely isolable (Kaufmann 1930, 23). This difference must be maintained in the logical relations by which the two generalities are translated. Let us consider the statements of specific generality (GS) and individual generality (GI), respectively:

(GS)'all colours have a certain brightness and a certain degree of saturation',

and

(GI) 'all men are between 40 cm and 1.5 m tall'.

In GS, the *all* is used to say that brightness and saturation are implicit in colour or, rather, that the presence of any colour (and the display of anything coloured) is incompatible with the absence of brightness and saturation. GI can instead be made explicit by the conjunction of

(GI*)'M_1 and M_2 and ...M_n are between 40 cm and 1.5 m high', and

(GI**) 'There are no other men than M_1 and M_2 and ...M_n'.

The logical relations that serve for GI are thus conjunction and negation, where the latter serves to delimit the scope of individuals. When this delimitation is not explicit, GI is incomplete, but this does not mean that it can be confused with GS. While, in fact, in this case, the domain of individuals of GI would be indefinite, for GS, the domain is made up of indeterminate individuals, i.e. variables, replaceable by any specimen that can be predicted to have, in the example, a colour. GI has a more or less definite extension, whereas GS is indeterminate in its extension. Moreover, the generality of GI is denumerable, restricted and distributive: one may not be able to say for how many individuals it applies, but it applies only to those individuals, not to others, and to each of those individuals. The generality of GS, on the other hand, is not denumerable, is not distributive, and there is no point in asking whether it is restricted or not: its exemplifications do not constitute its extension.

GS statements are general implications (Kaufmann 1931, 271, 279) and, therefore, since the variables involved are obtained by variation and substitution, they are analytic. In doing so, Kaufmann shows that he also inherits from Husserl (and indirectly from Bolzano) a conception of the analytic by substitution. If this is the case, however, the scope of substitutability of the analytic can be more or less wide, and depending on the extent of substitution, the analytic can be called formal or material, i.e. relatively analytic. The example of GS given earlier is analytic relative to the field of colour. A few years before Dubislav (1926) defined the relative analytic, which was also used by Carnap (1934, 44), another student of Husserl, Fritz London, had given a definition within the framework of a theory of deductive systems[6]. Although it cannot be proven that Kaufmann was directly acquainted with London (1923), an indirect acquaintance with him can be assumed through Geiger (1924), another text in which extensive use is made of Bolzanian concepts in a way that is not always stated. One of these concepts is that of incompatibility, which Kaufmann (1930) indicates as a characteristic logical relation of GS.

Kaufmann's use of incompatibility is ambiguous, and it is worth pointing this out at the outset because it will also be one of the themes of his confrontation with Carnap. Moreover, Kaufmann (1930, 84) acknowledges that it was Carnap who had given him a hand in improving the definition of natural numbers, in which the incompatibility relation is decisive.

Let us go step by step. *Incompatibility* is the name Russell (1925) gives to the *Scheffer Stroke* when it is understood as a negated conjunction ($\neg\varphi \lor \neg\phi$) and not as a negated disjunction, or *rejection* ($\neg\varphi \land \neg\phi$). Kaufmann (1930) uses it instead citing Geiger (1924), to whose elaboration the comparison with the young London, who in London (1923) had used incompatibility as an instrument of derivation, also contributes. What is relevant then is that for Kaufmann, incompatibility is to be read as a negated conjunction, but not between truth values. It should also be noted that incompatibility is primitive only for the language of GS, whereas for the language of GI, conjunction and negation are primitives. And this makes a big difference that Carnap does not necessarily mean on first reading.

The concept of incompatibility goes back to Bolzano (1837, § 159), who defines it as the relation that exists between the propositions A, B, C, D ... with respect to the representations i, j, ..., when no representation can substitute itself for i, j, ... making the propositions A, B, C, D, ... all at the same time true. It is only fair to point out that the representations are not the extensions of the propositions but their components and that the relation of incompatibility, if understood as the relation opposite to derivability, can be called an exclusion and a reciprocal exclusion in the specific case in which I have on the one hand 'A is as old as C' and 'B is three times as old as C' and on the other hand

[6] About London (1923) also Mormann (1991).

'A and B are together seven times as old as C' and 'B is as old as A and C put together'[7] .

Husserl uses incompatibility in pure grammar (Husserl 1900-01, 60), in the theory of abstraction (Husserl 1900-01, 233), in the distinction between non-sense and counter-sense (Husserl 1900-01, 334) and to illustrate the borderline case of non-filling or delusion (Husserl 1900-01, 632): in these cases, the incompatibility relation is used on propositional contents, meanings or concepts and functions as a contrast, a contrariety, of which contradiction is a restricted case.

Becker (1930) would then interpret incompatibility in the modal terms of the impossibility of the conjunction, i.e. introducing it as the equivalent in logic of the strict implication of the negation of the conjunction (i.e. in symbols $\neg(p \odot q) \equiv\sim (p \wedge q))$ and thus laying the foundations for axiom B (or Brouwerian) according to which from the impossibility of the impossibility of $(p \wedge q)$ does not result in the demonstration of $(p \wedge q)$[8].

In the logic-phenomenological tradition, influenced by Bolzano, which Kaufmann also falls partly under, incompatibility can be understood as a negated conjunction, with or without modalisation, and applies mainly to concepts or representations. Beginning with Bar-Hillel (1952, 1970, 59), the interpretation of Bolzano's incompatibility as NAND or as Carnap's (1942) mutual exclusion is also validated in the logicist tradition, but of course, with a true-functional reading.

In Carnap (1928a), on the other hand, there is never any need for *Unverträglichkeit*. Still, one finds in one case (§ 107) *Unvereinbarkeit*, translated in (Carnap 1967, 177) as *incompatibility*, as a fundamental concept of pure logistics, together with the validity of a propositional function for all its arguments, and in another (§ 147) *unvereinbar*, also translated in Carnap (1967, 227) as *incompatible*, to designate the discordant, but not contradictory at-

[7] Therefore, if we assume that C has 3 years, either it is not true that A has 3 years and B 9, or it is not true that A has 9 years and B 12, or both are not true. It only needs to be added that Bolzano distinguishes between derivability and consequentiality and that this distinction remains intact in the Husserlian theory of demonstration.

[8] One could also read the more recent revival of incompatibility in Brandom (2008, 123 ff.) as non-compossibility, i.e. "as a sort of conceptual vector-product of a negative component and a modal component", in these terms.

tributions of two psychical systems[9] . Beginning with Carnap (1927) and especially in Carnap (1934), the use of *Unverträglichkeit* is instead very frequent: I cannot claim that the comparison with Kaufmann dictated this, and it is more likely that it depends instead on a closer reading of Russell (1925) and related problems. It is not improbable, however, that the exchange with Kaufmann had some bearing, along with the *Bolzano Renaissance* in Dubislav (1931) and Scholz (1931)[10] .

The derivation of the negated conjunction from content incompatibility is not the only operation Kaufmann (1930) does to bring out the semantic content of logical forms. In Kaufmann (1930, 38-39), negation, conjunction and specific generality are derived in the following way:

1) Any states of affairs can be thought of as non-existent (or denied);

2) a plurality of states of affairs can be traced back to the existence or non-existence of certain common characteristics;

3) each object property occupies a certain position in a sequence of degrees of generality.

From 1) we derive the negation, since if and only if S is a state of affairs can it be said that ¬S, meaning that S does not take place and not that ¬S takes place; from 2) we derive the conjunction, since if and only if *x*, *y* and *z* can be said to be M, then $(Mx \wedge My \wedge Mz)$. If no other element in a given context is M, one can speak distributively of all M's (with the addition of the negation of 'none excluded'), thus forming a GI; from 3) we derive specific generality because if N is derived from M, and not vice versa, then N is incompatible with the non-existence of M, and not vice versa, i.e. M has a greater degree of generality than N (but not necessarily that it is more extensive or that N belongs to M).
In this respect, Kaufmann (1930, 39) adds that "negation and conjunction can be replaced, as Sheffer has shown, by the relation of *incompatible with*, if nega-

[9] It is worth emphasising that the difference between *Unverträglichkeit* and *Unvereinbarkeit* is, on the other hand, relevant for Bolzano and especially for Husserl (1900-01, 1929).
[10] In particular, Dubislav (1931, 451), after defining incompatibility as a relation between propositional functions (such that, given two functions fx and gx of the same variable x, these functions are incompatible if no value of x satisfies both fx and gx), adds in a footnote a remark very relevant for our purposes: 'for the assumptions that apply to Brentano, one must distinguish exclusion from both incompatibility and mutual exclusion, at least as far as systems of propositional functions are concerned'.

tion is indicated as incompatibility with itself". This addition clearly denounces the ambiguity I mentioned earlier.

Now, the most important application of incompatibility in Kaufmann (1930) is the definition of a natural number. The assumptions of this definition are:

the state of affairs in the numbering process,

the interpretation of this process as unambiguous coordination between objects (if any) and signs (if any),

the elimination of the temporal or succession character of this process,

the variability of the numbering order.

Therefore, when I say that the totality of numbered things is *n* (i.e. a certain cardinal number), I do not mean anything other than that the last of the numbered things is assigned the sign *n*, whatever the order of numbering was. Suppose there is only a pen, a hat and a book in a room at a given time that are yellow, or that there is only a painting made by my daughter, one bought at the junk shop and one painted by my friend, or that there is only Hans and Peter, what do I mean when I say there are 3 yellow things (Kaufmann 1930, 18), 3 paintings (Kaufmann 1930, 76) or 2 people (Carnap 1931/Hahn et al. 1931, 141)? Do I mean that 3, 3 and 2 are properties of the totality of the yellow things, paintings, and people in a room?

The next problem is to move from this idea of coordination of the numerical sign with the last of the things counted, whatever order the numeration followed, to the concept of the sequence of natural numbers understood as "a logical abstraction of the numeration process thought of as indefinitely continuable" (Kaufmann 1930, 82), i.e. as the feature that could be abstracted from the numeration process if we thought of it without determining its context of execution, i.e., in our examples, simply without room. If we made this abstraction, we would obtain a structure composed of incompatibility relations between variables or logical singularities. These singularities are neither individuals nor properties of relations between individuals, and so on. They are only the values that satisfy the incompatibility relations established by law, i.e. any elements of a structure defined only by incompatibility relations. Kaufmann's (1930, 84-85) and Kaufmann's (1931, 280) definition of a natural number is: "We designate as natural numbers the elements of the structure determined by the following stipulations and only by them:

1) there is one and only one element, with the presence of which the absence of any other element is incompatible;

2) for each element Z_m there is one and only one element Z_n with the presence of which the absence of Z_m is incompatible, while the presence of Z_n is also only incompatible with the absence of an element other than Z_m and Z_n , with the absence of which the presence of Z_m is also incompatible;

3) the relation determined by 2) between Z_m and Z_n is incompatible with an equal relation between another element and Z_n".

Kaufmann also specifies that the presence of an element means the application of a numerical sign, i.e. that a certain object is coordinated to a number according to a formal rule. Still, above all, the three stipulations that make up the definition of natural number coincide with Peano's first four axioms and make superfluous the fifth (complete induction), according to which "every property of 0, which also belongs to the successor of every number, belongs to all numbers" (Kaufmann 1930, 88). It is evident that the fifth postulate using 'every' and 'all' for specific singularities, such as natural numbers, is incorrect since the distinction between generalities, according to which I can quantify narrowly on individual generalities but not on specific generalities.

It is equally clear, however, that things are no better if I interpret the 'there is one and only one element' of 1) as a definite description and the 'for every' of 2) as a universal generalisation. Both (definite description and universal generalisation) were probably, as already mentioned, introduced by Carnap to make the definition of number more rigorous while exposing it to the circularity of the use of quantification.

Thus, if we take the schematic form of incompatibility $(\neg\varphi \vee \neg\phi)$ and substitute φ for the non-existence of Z_m (i.e. non-Z_m) and ψ for the subsistence of Z_n, we define the incompatibility between any two numbers at different positions (m, n) as follows

$$Inc_{(m,n)} =_{df} (m)(n)[Z_m \vee \neg Z_n]$$

and we can then formalise the description defined in 1) with:

$$\exists x((Zx \wedge \forall y \forall z(Zy \rightarrow (x = y \ \wedge x \neq z))) \wedge (Zx \vee \neg Zz))$$

using *x*, *y* and *z* as names for the numbers in positions *m*, *n* and *o*.

The only solution to plug the circularity hole would be to understand definite description and universal generalisation on arbitrary variables and not on individual variables and 1) as a derivation from the definition of the incompatibility relation, and this is to some extent how Carnap (1934) does it using free variables. If this were so, and if Z_m, Z_n and Z_o (as any number different from any other Z_m and Z_n), the sequence of natural numbers would be:

$$Zm \vee \neg Zn \vee \neg Zo$$

If, therefore, the natural number is the cardinal that corresponds to the element of an aggregate that is last in order (however the elements are counted), then such a disjunction should be the rule by which to apply the relation of succession between the ideal singularities *natural numbers* and with which to use the numerical signs, when these are univocally coordinated to some object. Being a disjunction, it also cannot be used to designate a finite or infinite totality. Finally, suppose I replace the notion of infinite with that of indeterminate and instead of speaking of unordered infinite sets. In that case, I speak of sets with an indeterminate ordering of the elements, I can also say that "the general element of a sequence is an indeterminate value, that is, the general form of a function with a variable for any natural number as its argument" (Kaufmann 1931, 286).

2.

The Husserlian position that forms the background of the distinction between generality that Kaufmann proposes from 1928 onwards and that I have just set out in the definitive form of Kaufmann (1930) and Kaufmann (1931) consists of three points:

a) the analysis of the general statements and comparison with the previous logic, as outlined between 1896 and 1911,

b) the analysis of abstraction in Husserl (1900-01) and

c) the Husserlian conviction about the explanatory precedence of cardinal over ordinal numbers and about the collective and non-distributive character of the generality of aggregates.

It is worth briefly reviewing them to understand Kaufmann's distances from Husserl, and also the contribution that the comparison with Carnap could make to this distancing.

a) The starting point for Husserlian reflections on generality is an unresolved comparison with the algebra of classes in Schröder's version from 1891-1893 (Husserl 1979, 3-92). The position then taken by Husserl was defined by Venn (1894, 475) as a middle way between "the rigid class, or extensive, interpretation, and the rigid notional, or intensive, interpretation" of predicative logic[11] . This is reflected in Husserl's (1891, 1979) belief that aggregates or classes are to be understood in a collective and not distributive sense. On this basis, Husserl distinguishes:

> totality (distributive) propositions, such as "many or all people get more than one", the (collective) class propositions "all men seek happiness" (Husserl 2001, 99) and propositions of laws, such as "conic sections can be cut by a line at a maximum of two points" (Husserl 2001, 171).

This results in a tripartition between: collective generalities, distributional generality and legal generalities, and in relation to the latter, the distinction between law and necessity (in the sense of law enforcement).

This allows Husserl to take a position with respect to two conceptions that emerged in late 19th-century German logic: the implication in general statements of a double negation (argued by Brentano and Sigwart) and the difference between empirical and indeterminate generality (argued by Sigwart alone). As to the first, Husserl agrees that in some uses of *all*, a none excluded is implied but notes that its explicit addition would unnecessarily complicate the source statement, that it is not clear whether that addition concerns the predicate and, above all, that the double negation only concerns restricted and distributive generalities. As for the second, i.e. the difference between empirical and indeterminate generalities, Husserl argues that it is an improper dis-

[11] Between the first and second editions of *Symbolic Logic*, Husserl had a brief correspondence with Venn, which revolves around the clarification of the meaning of *as such* in the statement 'an object of concept S is, as such, not an object of concept P' (Husserl 1994, 265-268). However, as we only possess Venn's letters and not Husserl's, it isn't easy to understand how, during the epistolary, Venn was able to understand Husserlian position better. However, it is difficult to make such an intermediate position formally unambiguous, as Husserl did not specify either then or later what he meant by the calculus of conceptual objects and what distinguished this calculus from the more traditional logic of intentions or concepts. One may suppose that he had found, in those years, useful tools to break the deadlock in the external (or referential) character of intentionality and in the internal character of incompatibility, combining which he could have obtained the extensive reference to individuals or objects, and the intensive relation between predicates. But this is, at present, only a hypothesis.

tinction for two reasons: because it is motivated in an exclusively psychological way and erroneously, and because it does not distinguish between two real types of generalities. In his opinion, firstly, the formation of the unity of a multiplicity does not take place by unifying mental representations or images, and secondly, the fact that in some cases, these images are all present. In others, they do not produce any difference in the logical form of generality. Indeed, it is not the prerogative of empirical generalities to be determinate and of non-empirical ones to be indeterminate: I could have several objects present before me and not be forced for this to unite them in a determinate and distributive generality: I can say 'all the apples on the table', 'each apple…' or simply 'the apples…'. The real difference that underlies Sigwart's theme is rather a difference between propositions to which generality can be attributed, i.e. between propositions that presuppose the existence of individuals and those that do not (Husserl 2001, 168). The latter are legal generalities that cannot be understood in a distributive or collective sense. Finally, even if the latter were to assume the existence of their objects, they would do so by means of surrogate propositions, as when before stating a property of conic sections one would say "assumed that the conic sections are…" (Husserl 2001, 168, 176; Husserl 2003, 163).

These legal generalities correspond to the specific generalities of the *Second Logical Investigation* to which Kaufmann often refers. In (Husserl 1900-01, 116), Husserl distinguishes:

singular propositions	*universal propositions*
individual: 'Socrates is a man';	individual: 'all men are mortal';
specifications: '2 is an even number';	specifications: 'all analytical functions are differentiable'.

Before turning to the theory of abstraction, with which Husserl basically justifies this classification of propositions on the basis of the objects on which they are based, it should be added that only individual universal propositions admit quantification. In contrast, individual singulars do not bear an existential generalisation (because by not pronouncing on Socrates I cannot even replace him with a variable). Specific ones (singulars and universals) only admit quantification of their applications, i.e. only when we pass from a proposition expressing a law to a proposition expressing the necessary condition for the application of a predicate to a domain of individuals.

b) If it is easy to indicate the polemical referent of Husserlian abstraction theory, it is more difficult to define it unambiguously. In fact, his critique concerns the main forms of the long tradition of abstraction on a sensualistic basis, from Locke to Mill: it targets both the aporias arising for the conception of any object, or in general, and those concerning the generality of predicates (qualities, properties and relations). The two themes (objects in general and generality) are different. Still, Husserl believes they can both be addressed by a theory of abstraction that perfects the Humean *distinctio rationis* and establishes a clear link between the perceptual basis of real individuals, substitution or variation and ideation. In doing so, Husserl could be said to be taking a conceptualist position if conceptualism is understood as Quine does (1947, 160). However, two drawbacks remain: the first is that for Husserl, conceptualism is Hamilton's in controversy with Mill, and the second is that even in Quine (1987, 68), the conceptualist is the one for whom every abstract must be specifiable. And if this, as we shall see, is true for Kaufmann, I am not sure that it is equally true for Husserl. Certainly, he is convinced that "in the real world, just as one cannot find numbers or triangles in general, neither can one find possibilities" (Husserl 1900-01, 120), that the ontology of the perceptual world is made up of concrete individuals, but I do not think he shared such a tight constraint of instantiationality on admissible generalities. There is no doubt that 'the red' is a grounded object and that it is grounded in some red object (perceived, remembered or imagined). But 'red', like the postulate of parallels and virtue are impure concepts, i.e. an intermediate stage between the typification of the colour of some fabric and the ideality of logical or mathematical propositions. If that constraint of instantiation were valid on all three of these levels, then of all three, we could have an intuitive translation (in an immediate or mediated manner), and there would no longer be any room for symbolic thought (which is not intuitively satisfiable) nor any difference between intuitive and symbolic. And yet this difference (which Husserl formulates in various ways over the years) is indispensable precisely for the treatment of logic and mathematics. With this, I do not deny that in Husserl, one can find all the aporias of a neo-Aristotelian logic, in which *what there is* is individual and what I can say about it is, however, general. Finally, substitution and iterability, as

Weyl (1918) would rightly recognise, are characteristic operations not only for the phenomenology of logic but also for that of experience. Only by thematising, or nominalising, a proposition can one form a state of affairs; only by thematising the colour of an object or the sound of a melody can one understand the incompatibilities between colours or between sounds, only by thematising one's current experiences can one think of an 'I'. Even if I recognise the symbolic, and therefore non-intuitive, character of iteration and substitution, I do not dismiss them of their validity.

c) Finally, I come to the connection that exists for Husserl between the logical priority of cardinals and the collective, i.e. cantorian-like, interpretation of classes. In (Husserl 1891, 175-176) – a text that Kaufmann actually quotes very little, and in any case not in this respect – Husserl criticises the nominalism of Helmholtz and Knonecker and their belief that ordinals constitute the natural starting point for the subsequent development of the concept of number. There are two main reasons for this criticism: because the cardinal number does not designate the character of a series when its order varies, and because the value of the concepts of class and aggregate for the formation of the number concept is thus lost sight of. Moreover, this allows Husserl to distinguish classes that have no order between elements from those whose order is left undetermined. The comparison with Frege should ultimately be brought back to this level as well: the problem is not that for Husserl, the number directly designates the aggregate and not the concept of which the aggregate is the extension, but that aggregate or class are not understood distributively, but collectively so that the aggregate or class count as ideal units.

It seems clear to me, therefore, that the three points of the Husserlian position correspond by contrast with as many points of the Kaufmannian position, namely:

a') with the notion of empirical generality, which, compared to a) is much smaller and similar to some positions of the old logic,

b') with the interpretation of the abstractness of properties exclusively in terms of tropes (i.e. properties instantiated by real individuals), and

c') with the declaration of the precedence of the ordinal over the cardinal, set out in Kaufmann (1930), as a direct consequence of b') and in clear opposition to c).

3.

Carnap notes in his diary that he met Kaufmann for the first time on 18/6/1927, exactly the evening before he made the acquaintance of Wittgenstein (Carnap 2022, 339). Carnap remembers talking to Kaufmann about the constitutional system, i.e. the *Aufbau* and the vision of essence, commenting ironically that his interlocutor admitted that one could get by with a minimum of data. Only a few months later, on 1 July, Carnap reported a second encounter, this time a fruitful one on Kaufmann's manuscript (1930) (Carnap 2022, 371). The first real exchange took place with the sending of the manuscript of the *Untersuchungen zur Axiomatik*, which Kaufmann commented on in a letter dated 7/12/1928 (KN 8081-8082). In the *Untersuchungen* that were not published, except for a brief *Bericht* (Carnap 1930), Carnap continued work begun in Carnap (1927) on a problem inherent in the application of the constitutional system of *Aufbau* to the foundation of mathematics.

Carnap (1927) distinguished between proper concepts introduced by explicit definitions in a broad sense (including definitions in use), such as real and formal concepts, and improper concepts, whose definitions are implicit and derive from the axiomatic system (henceforth AS). Moreover, improper concepts can be independent (such as Peano's number) or non-independent (such as Hilbert's point), and respectively independent monomorphic (for a complete number series) and polymorphic (for a single number), or non-independent monomorphic (for a system of six fundamental concepts) and polymorphic (for a single point or a complete class of points). One could thus reduce the difference between proper and improper concepts to the fact that "the principle of the excluded middle, which holds without exception for proper concepts, does not hold for all improper concepts and not only for polymorphic ones" (Carnap 1927). Carnap recalls that the same difference had been defined in terms of completeness and incompleteness in Weyl (1926), of definiteness or indefiniteness of decision in Becker (1924), of categoricity and disjunctiveness in Veblen (1904) and, significantly for our theme, of singularity and generality in Couturat (1905). The question returns in Carnap (1928b) in his attempt to construct a categorical (i.e. monomorphic), decidable (for which the third exclusion applies) and non-bifurcable axiomatic system. An axiomatic system is bifurcable (as, for example, geometry with or without the fifth axiom), when, given a function g, that system with reference to g is satisfied by both $f \wedge g$ and by $f \wedge \neg g$ (Awodey, Carus 2001, 157).

Carnap's (1927) and Carnap's (1928b) problem is a version of the Hilbertian problem of the definiteness of a propositional variety or an axiomatic system, with the typical logicist-Fregean concern not to leave (improper) concepts up in the air and thus with the intention of justifying their contents and not their, so to speak, syntactic position. It should be noted that Hilbertian completeness was inaccurately translated as definiteness in Husserl (1901), later becoming his workhorse, with continuous applications not only in the philosophy of mathematics but especially in the theory of science up to Husserl (1929). The fundamental relation of Husserlian definite variety theory would have been that of incompatibility, derived from Bolzano (1837), and applied to representations, propositions and truths. In Husserl (1900), there is no mention of *definite Mannigfaltigkeit*, and when *Mannigfaltigkeitslehre* is needed, it has a Cantorian-Leibnizian sense (Husserl 1979, 250 ff.). In contrast, it is discussed in *Ideas I* (Husserl 1913, 151 ff.) somewhat hastily, as if it were a settled issue. It is worth noting that Kaufmann (1921, 39) objects to the Husserlian optimism of *Ideas I* regarding the finite definability of all varieties that although the number of concepts in an axiomatic system can be considered finite, the number of derivable propositions cannot be said to be finite. Since they are not finite, it cannot be decided which of them can be contradictory.

Yet from Kaufmann (1930), we deduce not only Carnap's (1928b) reading but also his appreciation, a symptom of a clear change of opinion on the matter. In his letter of 7/12/1928 (KN 8081-8082), Kaufmann vindicates his radical finitism but reproaches logistics for failing to solve the problems raised by the transfinite. More specifically, Kaufmann imputes four main instances of conceptual inaccuracy to Carnap: to have confused species and totality of individuals, not to have used the incompatibility relation in the construction of a theory of natural numbers, to have overlooked the symbolic character of the construction of the transfinite and to have replaced, out of fear of metaphysics, the relationship between language and the world with that between logic and language, and then tried to remedy the proliferation of objectless propositions with the Russellian theory of types.

This is probably the first occasion when the difference between specific and individual generalities emerges, so much so that Carnap reports on 16/12/1928 of his long meeting with Kaufmann and how important his distinction between species and collective concepts seemed to him (Carnap 2022, 384). From the same day is a manuscript (RC 028-26-11), in which

Carnap reports in his way on the distinction presented to him by Kaufmann. On the one hand, there would be the collective-individual concepts, such as
x=Brown or John or James
x is a piece of furniture in this room
x is an inhabitant of London;
on the other, species concepts such as blue and natural numbers.

Whereas statements with collective concepts can be confirmed with a *one-off payment*, universal statements on species concepts would be nonsensical, as when I say 'all blue things are square' and I cannot know whether I have proved them all even if there were only five blue things in the whole world. This clarification has as its stated polemical objective Russell, as Carnap again reiterates (25/5/1930, RC 028-26-07), namely his conviction that "for arithmetical existence and that is to operate sensibly with it it is enough if there are enough things". Carnap then proposes two formal translations for collective statements: one with a restricted universal

$$\forall x(x\ is\ a\ furniture\ in\ this\ room) \rightarrow (x\ is\ blue)$$

and one in terms of inclusion in a class

$$(furniture\ in\ this\ room) \subset (blue).$$

He also adds another difference concerning the concepts of blue species and natural numbers. While the statements 'x is blue' (because x is some locally determined individual) and 'all the furniture in this room is blue' (because here a collective concept recurs: 'all the furniture…') would be acceptable, 'x is a natural number' and 'all prime numbers are natural' would not be, because a number, such as 5, is not an individual but a logical singularity. Carnap finally points out that while the individual is never fully describable, "the logical singularity is nothing but a carrier of certain incompatibility relations".

The reference to incompatibility relations leads back to Kaufmann's letter quoted above and its Husserlian-Bolzanian paraphernalia. At the time, Kaufmann argued that the misunderstanding of the symbolic character of the construction of the transfinite also stemmed from the fact that the symbolic translation of incompatibility in the Sheffer Stroke did not help to understand what incompatibility meant and what it existed between. Also linked to this was the confusion between mathematical knowledge and mathematical symbolism, as well as the desperate attempt to rescue the relationship between language and the world through type theory.

The exception, however, is well understood. Certainly, very much present in him is the somewhat naive and reactive rejection of the semiotic dogmatism (as he would define it in a letter of 22/7/1929, to which I will return) prevalent in the Vienna Circle, i.e. the blind faith in the virtues of symbolic language, and the consequent condemnation of any argument not expressed in that language[12] . Nevertheless, when Kaufmann claims that in Carnap (1928b), there is no clarity about the pseudo-symbolic construction of the transfinite, he means something else. And it is something that concerns, first, the application of incompatibility and then his conception of the symbolic. In this passage, and then more clearly in Kaufmann (1931), Kaufmann endorses the interpretation of incompatibility as a negated conjunction and thus as a contrariety but denies that it should be applied to the truth values of propositions, whatever they may be because it should rather be applied to conceptual contents.

Returning to Kaufmann's exception, then, the first move (that of bringing attention back to incompatibility and using it in the Husserlian-Bolzanian sense) preludes the next move: to consider *p* and *q* (between which incompatibility subsists), as logical singularities, i.e. as conceptual contents and as pieces of a language in relation to the world, and not as symbols of logical language in relation to mathematical language. One might wonder why Kaufmann's naive appeal to recover the relationship between language and the world did not throttle the confrontation with Carnap before it was born.

I believe there are two answers. First, I believe that Carnap was not entirely insensitive to the phenomenological attempt to justify what in Carnap (1928a) is the extensionality thesis and in Carnap (1934) will become the extensionality hypothesis. Proof of this is the fact that in the literature at the end of § 180 Becker's (1924) accessibility principle is used to support the thesis of decidability of all questions and as a criterion for the demarcation of science. During the paragraph, it had been sufficient for Carnap to use Schlick's (1918) verificationist principle of meaning, to which he added with some mischief the reference to the valuable but hardly comprehensible Wittgenstein (1922). The reference to Becker, however, does not only serve to say, as Carnap often does ecumenically, that that same principle is shared by positivists

[12] And it should be remembered that between June and July 1929, Kaufmann's refusal to sign the brochure *Wissenschaftliche Weltauffasung* (KN 8088-8090) was consummated.

and idealists as if to say by all the forces of the philosophical parliament. In § 180, there is one of the few uses of the Russellian type theory, which Carnap had said in §29 that he could not go into and to which, however, the thesis of extensionality and the difference between objects and quasi-objects is linked. Whereas in §30, Carnap had emphasised that his spheres of objects are Russell's types used for non-logical concepts, in §180, he specifies instead that, to avoid pseudo-propositions, it is necessary to establish whether in the series of words under examination, there are any that are meaningless or whether their meaning is incompatible with the context of the proposition. In the case of logical language, then, as there is no need to look at the meaning, it suffices to pay "attention to the type of the sign (which corresponds to the sphere of the object)". Add to this that decidability is a *hapax* in Carnap (1928a). The decidability that derives from the principle of transcendental idealism is, however, limited to the accessibility and actuality of the states of affairs on which the propositions are based and is bound by the intentional relation. Thus stated, this principle cannot fit into Carnap (1928a). Yet the position in which it is embedded – between decidability, type theory and discrimination between propositions and pseudo-propositions – well explains the interest it arouses in Carnap between 1928 and 1934, just as it explains the interest in the distinction between generalities that is closely linked to that principle, as will be seen shortly.

The second answer to the question I posed earlier lies in the way Kaufmann expounds his idea of the relationship between language and the world by means of a certain analysis of language (as a pattern of coordination between symptoms, signs and objects) and a certain logical definition of the world, made up of individuals and properties instantiated by individuals. On 5/4/1929, Carnap (2022, 399) notes that he had read Kaufmann's manuscript (1930) for the whole day and found "the reference (in detail justified) to the double meaning of *all* instructive and significant". He then thanks Kaufmann for making it clear how useful the results of Husserlian investigations can be but does not spare himself to add that he leaves it open for the time being whether it is really necessary to do so. Nor is he generous with Viktor Kraft's lecture on Husserl's FTL (21/2/1930), which Carnap (2022, 453) calls "boring". Yet the conversation about the two types of generalities is repeated on 3/6/1929 (Carnap 2022, 407) and also extends to the distinction between existential and general quantification, understood respectively as a series of disjunctions and conjunctions. This distinction is described as contestable in Carnap (1929, 14), where in the literature, he cannot cite Kaufmann (1930) but Weyl (1926) and Ramsey (1926). Shortly afterwards (on 29/7/1929, KN 6089; RC 028-26-09), Kaufmann sends his comments on Carnap (1928a).

Kaufmann's objections are grouped into three main points: the interpretation of Russell's dictum (according to which a scientific philosophy must proceed to the replacement of inferred entities with logical constructions), the definition of pure knowledge of so-being and thus of generality (an issue that includes the translation of properties and relations into classes and the difference between objects and quasi-objects), and the possibility of a classification of the sciences based on the use of structural descriptions. The main argument, however, is the second; linked to it are:

a) a finitistic and constructivist conception of mathematics, which includes the rejection of Peano's 5th axiom and Russell's principle of abstraction (which Kaufmann and Carnap had already discussed on the basis of Kaufmann's manuscript (1930) and which is therefore only marginally touched upon by these remarks),

b) a certain idea of generality and the unambiguous relationship between general and individual, thus

c) the interpretation of the grounding of the grounded relations of *III Logical Investigation* and

d) the distinction between two types of reducibility, and finally

e) the themes of *Verdeutlichung* and

f) of the difference between non-sense and falsehood of an assertion.

In the letter accompanying his *Remarks*, Kaufmann begins with the link between clarification, knowledge of properties and the realism dispute: realism becomes a pseudo-problem only when one forgets that the only other form of knowledge apart from empirical knowledge is that which derives from clarifications or theoretical distinctions and that these theoretical distinctions are operations through which the so-being, the qualities instantiated by empirically known objects, are brought to the fore. Now, assuming that what is real is equivalent to the empirical individual, to what has spatial and temporal localisation, no matter whether simple or complex, while so-being is the general, or the only general admitted by Kaufmann's empiricist finitism, then the dispute over realism is at the same time a dispute over universals. Indeed, from his point of view, discussing universals becomes the best way to resolve the dispute over realism. Could it not be said that the frequent discussions in Vienna and Cambridge, as well as between Kaufmann and Carnap, on type theory and the axiom of reducibility is the revised and correct way in which scientific philosophy approaches its dispute over realism through a dispute over universals? This is undoubtedly Kaufmann's view and is the reason why he sees a connection between abstraction theory and the philosophy of mathematics.

In his view, both the phenomenological theory of abstraction and the logicist replacement of properties with classes of objects (of which properties can be predicated) serve to overcome the circularity in which traditional empiricism's idea of abstraction ended. And once the theory of abstraction is identified as the common target, the road is downhill. If the individual is not indivisible, but that which can be shown to have access to and can therefore be claimed to be real, and if the general is dependent on the individual, is a grounded object, a quasi-object, a characteristic of it that can be distinguished by clarification, then the passage to the extensionality thesis and the class labyrinth is superfluous. A phenomenological theory of abstraction is sufficient. It is necessary to cure Russell's (1925) defects with the medicine of *II Logical Investigation* (22/7/1929). Admittedly, Husserl also used the notion of object in a broad sense. Thus, the overcoming of the Fregean distinction of object and concept in §5 of Carnap (1928a) is acceptable for Kaufmann. Still, the idea that the boundary between individual and general concept is displaceable is not, as will be reiterated in §158. Furthermore, it is an equivocal use of properties that Carnap makes in the examples in §10, in which he equates descriptions of properties of a historical individual (his dates of birth and death) with those of conic sections or curves, which are not individuals, but ideal singularities and thus already properties.

As for the Russellian motto, it should be rewritten as follows: a scientific philosophy must replace inferred entities with logical clarifications. It should be remembered that "a distinction must be made between reducibility on the basis of theoretical clarification analyses and reducibility to coordination relations on the basis of empirical criteria" (RC 028-26-09, 6).

It is worth noting that Kaufmann wrote this last remark with reference to the realist language of states of affairs in § 47 of Carnap (1928a) and that it distinguished individual states of affairs expressed by assertions and general states of affairs expressed by propositional functions. The latter distinction is also essential for the development of Kaufmann's thought in his American phase. It derived from Russell's distinction between assertion, proposition and propositional function and that between apparent variable and real variable. Still, it would find confirmation and reformulation first in Husserl (1939) and then in Dewey and especially in Lewis.

In a letter dated 10/2/1931, Kaufmann makes explicit a circumstance that is not only biographical. As *Notebook 25* attests, he was reading Carnap (1928a) and Husserl (1929) together, both of which he reviewed in 1930 and 1933. "To so-being", Kaufmann writes, "can thus be linked to the issue of clarification, which is to establish how many degrees of freedom the quality in question has. In FTL, Husserl analysed this theme of clarification even more poignantly than in *Ideas*, and there, he no longer thinks of it as an intuitive vision that transcends experience". In this way, Kaufmann reassures Carnap that the distinction between generalities, which in Husserl is based on the analogy between sensible and categorical intuition, has been freed from the hypothecation of a hardly digestible idea of non-sensible intuition. Nonetheless, Carnap remained wary of the *Verdeutlichung* and the fact that it represented the only other type of knowledge besides empirical-individual knowledge, and this will be evident in the comparison of the preparatory manuscripts to Carnap (1934), not least because of the affinity that not only Carnap but also Kaufmann suggests between *Verdeutlichungen* and Wittgenstein's *Erläuterungen.*

Kaufmann, however, derives not only this from his reading of FTL but also the idea that logic has a bond with the world and that if he takes this for granted, if he does not put it to the test, logic becomes a positive science not unlike physics or history. Still in 1931, and still thinking of Husserl (1929), Heyting, in the Königsberg discussion, reproaches Carnap rightly that "the logicists do not want to accept that in the construction of mathematics, the concept of the world is already used" (Hahn et al., 1931, 147).

It is, therefore, inevitable that, especially in the absence of a theory of assertion that would only come later and that would anchor realist language in pragmatic-linguistic conditions, Carnap sees in Kaufmann promising positions for Carnap's foundationalist project (1928b) and (1930) and that, when this project runs aground, his attitude towards Kaufmann also changes. The problem is that Kaufmann also understands his position in the same way, as is shown by the frequent references in the letter and in the remarks mentioned above, but also in Kaufmann (1931), to the discussion he had with Behmann

(1930) on the possibility of giving a constructivist, decidable version to the indirect proof (and thus on the preservation of the whole of classical mathematics within a constructivist framework). 1931 would not only be a decisive year for Carnap (1934) but also for the Kaufmann-Carnap confrontation and Carnap's refunctionalisation of Kaufmann's restricted realist language.

Before this happens, however, on 23/5/1930, Carnap reciprocates with some remarks on Kaufmann (1930), written in view of the review. Before examining these remarks, it is worth noting that there is a consideration in them that does not appear in the review (Carnap 1930). Commenting on Kaufmann's use of the distinction between generalities also to address the problem of decidability, and in particular Kaufmann's (1930, 189) claim that the insolubility of the decision problem is ruled out, or that in any case it can be shown that a problem is insoluble, even if its insolubility cannot be demonstrated, Carnap writes that "it is not ruled out, however, that the insolubility of an arithmetical problem can be demonstrated" (RC 028-26-07, 5). Significantly, this consideration was not later included in the review. In any case, the problem of decidability is not central in Carnap's remarks. Rather, the following issues are isolated in them:

a) the tracing of the pseudo-problem of the actual infinity (in the theory of transfinite numbers) to the error of having used mathematical symbolism out of its scope, of having made excessive use of symbolism;

b) the assumption of ordinal numbers as a starting point and thereby a double stance in relation to Brouwer: agreeing with constructivism and disagreeing with the use of the concept of time (van Atten 2007, 38-39);

c) the renunciation of Peano's postulate of complete induction, with a procedure akin to that of Fraenkel's axiom of limitation;

d) the treatment of rational numbers; and especially

e) the link between the distinction between generalities and the concepts of set and class.

Carnap shows that he particularly appreciates the definition of natural number sub c), which is also found in Kaufmann (1931, 280) and makes extensive use of the notion of incompatibility. In his notes of 5/4/1929, Carnap had already agreed with Kaufmann on this point and had also helped him to refine the definition, as I have already mentioned.

It is thus better understood how the incompatibility relation is an essential tool to put the distinction between generalities into practice and why Kaufmann (1931, 290) understands it as the result of an immanent critique of the use of signs. Without incompatibility, nothing could be derived from specific generalities.

Concerning the distinction between generalities and thus e), Carnap notes how aggregates can be understood as enumerations of objects and classes as expressions of laws or properties, and it seems that this annotation also serves to confirm the distinction made in Carnap (1928a) between classes and totality. It follows that one could analyse statements about infinite sets as general implications drawn from laws of formation. This, however, urges Carnap in a different direction from the one indicated by Kaufmann (1930) and also different from the direction followed in those years by Carnap himself, but consistent with Kaufmann's (1931) immanent critique. Suppose the foundations of set theory are rotten. In that case, if it is futile even to attempt to save them with axiomatisation, and if the pseudo-problem of infinity arises from an abuse of mathematical symbolism, then only work on syntax can do in this context what type theory has done with antinomies. It is not enough, Carnap points out caustically on 30/6/1930 (RC 028-06-08), to reflect on meaning. Now, although Awodey, Carus (2001) have noted that in the review, which derives from the *Remarks* I have just considered, there is one of the earliest examples of a conscious distinction between logic and metalogic, between construction and syntax (understood as a set of "metamathematical instructions for the operations of the calculus" (Carnap 1930)), this is not how Carnap (1931) will apply the distinction between generalities proposed by Kaufmann.

Indeed, in Carnap (1931), we find the main application of the distinction between generalities. His decisive question is: "is it possible to maintain Ramsey's result without sharing his absolutist conception?" (Carnap 1931, 102). Ramsey's absolutist conception (1926, 39) could be expressed as follows: all propositions are truth-functions of their elements, and that is enough. Even if I cannot construct these elements, they will have some truth-value, and so they are needed in the function. Thus, having distinguished the sphere of functions of individuals as functions in extension from that of functions of functions as predicative functions, a predicative function is a truth-function of arguments which, finite or infinite in number, are either atomic functions or propositions. As for the meaning or truth value of a proposition, Ramsey (1926, 48) explains that it corresponds to a certain number of realised possibilities.

Ramsey's result, on the other hand, is "the limitation to the simple theory of types and yet the possibility of defining mathematical concepts, and particularly the theory of real numbers. This result would be achieved if we, as Ramsey does, considered non-predicative conceptual formations acceptable. But can we do this without accepting Ramsey's conceptual absolutism?" (Carnap 1931, 102). That is, can we renounce the axiom of reducibility, even without accepting that the set of properties already exists prior to their description by definitions? Carnap believes he can do so precisely because of the difference between generalities.

Russell (1908, 248) introduces the axiom of reducibility (AR) in the form

$$(\exists f):.(x)\varphi x. \equiv .f!x$$

i.e. for any argument (of any type) of a propositional function (of any order), there is at least one equivalent predicative function, a predicative function being a function of the order immediately following that of its argument.
Feferman (1998, 261) formulates AR in the language of informal set theory as follows:

$$\forall X^{(j)}\exists X^{(o)}\forall n[n \in X^{(j)} \leftrightarrow n \in X^{(o)}]$$

in which the indices denote orders and the order o is the elementary order of the predicative functions.

Russell (1910) already considered it difficult to maintain that AR was self-evident, even if he did not go so far as to recognise, like Ramsey (1926, 65), that it could only be true for a *happy accident*. His retention stemmed from the belief that it was the only way to avoid the paradoxical consequences of the ramified theory of types, such as infinite empty sets and infinite total sets. Once it had been established not only that every propositional function could not have an argument of the same type, i.e. of the same scope of significance, but also that the same argument could occur in functions of different orders, it was necessary to reduce these orders to the most elementary one. Ramsey (1926, 48) also considered the ramified theory of types dispensable (and thus AR with it) because it was only useful for epistemological or semantic paradoxes (such as Epimenides) that are not relevant in the foundation of mathematics. In contrast, it was not useful for mathematical or syntactic paradoxes (such as the class of all classes) for which the simple theory was sufficient. Russell (1925) had also tried to replace AR through the principle of partial extensionality, regarded as the non-universally true assumption that "the matrix $f!\,(\phi!\,\hat{z})$ always originates from some stroke function F(p, q, r,...), by replacing part of p, q, r, ... or all of p, q, r, ... with $\phi!\,a$, $\phi!\,b$, $\phi!\,c$"[13] .

Why, then, does Carnap (1931) think he can use the distinction between generalities to eliminate AR? The issue is that the validity of a general statement cannot be proved on the set of individual cases. Take the non-predictive definition of the inductive property:

$$Ind(x) =_{Df} (f)[(Erbl(f) - f(0)) \rightarrow f(x)],$$

i.e. x is an inductive number, if for every f that is a hereditary property and 0 enjoys this property, then x will also enjoy the same property and try to prove its application to x=2, then

$$(f)\ [(Erbl(f) - f(0)) \rightarrow f(2)].$$

[13] On this, see Parrini (1977). Weyl (1918, 36), too, arguing the difference between proper and improper concepts, i.e. predicative and non-predicative, and between individual, general and universal, contrasted his restricted procedure (limited to proper concepts) with type theory and AR. Adhering to this procedure, which limits the use of the concept of existence only to the fundamental categories (natural and rational numbers) and not to systems of properties and relations, is not, however, obeying a command, but only obeying a law (of conceptual formation). "In science' there are only 'laws' and not 'commands'. Therefore, there is no need at all to prohibit the use of the expression 'there is' in reference to objects that do not belong to the fundamental categories. It is possible (and acceptable) to follow an extended procedure: it is sufficient that those who do so, do so, however, without running into circularity" (Weyl 1918, 24). It is fitting to add that Weyl (1918) is the most faithful extension of Husserlian philosophy of logic for the following reasons: for the treatment of existential propositions, for the formation of generalities (through existence and without negation), for the consideration that the individual description of a finite set is only one case of the description according to law or general, and especially, for how he extends the restricted procedure through iteration and substitution, leaving only existence exclusively to the basic categories. The only disagreement with the Husserlian model is that Weyl (1918, 68) leaves the priority between ordinal and cardinal undecided. This brief digression on Weyl (1918) was necessary not only because it is a source common to Kaufmann and Carnap but above all because of the refusal to understand the narrow procedure as an obligation to be adhered to, a refusal that seems to me very similar to Carnap's (1934) refusal for any morality in the logic with which he introduces the principle of tolerance and with which he marks the transition from his narrow (realist, finitist and Kaufmannian) language to the broad language of L II. Cf. Uebel (2009).

However, when we handle non-predicative definitions, we know that the series of their individual cases is an infinite totality. "The idea that it is necessary [to go through the individual cases] derives from the confusion between 'numerical' generality, which refers to the given objects, and 'specific' generality (Kaufmann 1930). Specific generality is not established by examination of individual cases, but from certain determinations, one can logically derive others" (Carnap 1931, 103). In the case of the inductive property, Carnap continues, if we consider its definition to be a statement of specific generality, then it means that from the determination 'it is hereditary and belongs to 0' we can derive the determination 'it belongs to 2', through two steps: the obvious derivation of f(0) from (*Erbl (f)* - f(0)), and, the introduction of the definition of the hereditary property *Erbl*(f) $=_{\mathrm{Df}}$ (n) [f(n) → f(n + 1)]. By then substituting 0 for n in the definition of inheritance, we obtain that the inductive property is valid for 1, and by substituting 1 for n that, the same property is also valid for 2, and so on. Therefore, "if the general validity of a statement for any property only means its logical (or rather: tautological) validity for an indeterminate property, then we can also accept non-predicative definitions as logically acceptable. If, however, a property is defined in a non-predicative way, the decision on its subsistence or non-subsistence in a single determinate case may be difficult or even impossible if logic does not present a decidable system (*entscheidungsdefinites*). By no means, however", Carnap concludes, "is this decision in principle impossible due to non-predicativeness".

It should be noted that Carnap's application of the difference between generalities is *faithful*: because it recognises that only GI is reducible to individual cases and not GS and because it regards GS as a derivation of some determinations from others; is not *faithful* instead: because he calls it a tautological derivation, and especially because it does not use incompatibility as a derivation relation; finally, this application is crushed by the problem of decidability, i.e. the exclusion of undecidability in principle of an AS, which, although present, as we have seen, in Kaufmann (1930), is not the reason why the difference between generalities was introduced. If, on the other hand, one restores incompatibility as a derivation relation and uses GS as a new rule of analysis that makes it possible to translate certain universal quantifications not as conjunctions (with or without negations that close the domain) but as incompatibility relations, the situation changes: GS *serves less in the logical foundation of mathematics, but much more in the logical clarification of mathematics.*

In the same year as the Königsberg Congress, Kaufmann threw himself headlong into a failed project with Behmann (Mancosu 2010) and continued to work on an essay that was to have appeared in Husserl's *Jahrbuch* and remained unpublished. Kaufmann sent one of the last versions of his manuscript to Carnap, and the latter drafted his very harsh *Remarks* on 29/11/1932 (RC 028-26-01), even though almost a year earlier (on 14/12/1931) he had written to him, "I shall be very interested in reading the radicalisation of your constructivist thesis". On 11/12/1932, Kaufmann and Carnap discussed these *Remarks* and the fifth chapter of Carnap's manuscript (1934), which was then, as we shall see, entitled *Semantics,* and Carnap (2022, 571) notes: "we do not agree at all, but we discuss very calmly and understand each other".

In the early pages of Kaufmann (1932, 3209), we read that "language is not a system of acoustic complexes ('words') and their configurations ('sentences'), but a system of rules that correlate these complexes and configurations with certain contents of thought". Thus, Kaufmann repeats a conception of language as a pattern of coordination between symptoms and words that he referred to Carnap several times, commenting on Carnap's (1928a) and especially Neurath's physicalism. But he does so using exactly the terms of the first part of the (then *Metalogic) Syntax* on which Carnap was working. In fact, the latter notes in the margin of the quoted sentence, "nobody understands this". The same is true when Kaufmann (1932, 3210) raises the central question for him of "what does an assertion about something mean?" and Carnap comments: "this is the typical phenomenological attitude... one should rather ask how we want to apply the expression *assertion about...*". Referring then to Kaufmann (1932, 3252), Carnap notes that the whole discussion on the formation of non-predicative concepts is just a tilt at windmills because the wrong definition of Fraenkel (1928) is used. According to Carnap, in the section on the resolution of antinomies, the problem is not even seen. And he concludes lapidary: "in general about the manuscript I have very little to say. The difference between views and questions is too great".

More than one thing, however, escapes or does not interest Carnap from Kaufmann (1932), beyond the glaring difference between the questions "what is an assertion?" and "how do we want to apply an assertion?", which is the difference between two different conceptions of language, but above all of how one can talk about language and what one can say about language. This is a central difference in the second phase of the Carnap-Kaufmann confrontation, the one that runs through Carnap's (1934) drafting, a difference that rests on a profound disagreement over intentionality and reference, i.e. the intentional character of realist language, and once again on the function of reflection and clarification.

However, in Kaufmann (1932) one finds:

a) the realisation, drawn from FTL, that the analysis of the morphology of language puts out of play not only the truth but also the content of truth, its semantic amount;

b) the conviction that logical syntax, derivable from FTL even more than from *IV Logical Investigation*, is as different from Wittgenstein's (1922) pure grammar as from descriptive syntax, of which he knew from Carnap's (1934) manuscripts;

c) the idea that the link with the world that FTL's syntax has, through the theory of syntactic and non-syntactic cores, cannot be translated into Wittgenstein's (1922) conception of language as an image of the world;

d) the design of a minimal syntax with two postulates:

d.1) that every state of affairs consisting of linguistically fixed elements (the syntactic cores) must be expressible by means of a syntax established once and for all;

d.2) that every statement formed in compliance with the rules of syntax, or principles of forming and transforming provable statements or formulae (Kaufmann 1932, 3218-3219, 3225-3226), must be semantically assessable[14];

e) the expression, never so clear, that "only a body or a person (only a concrete physical or psycho-physical object) can be the subject of an assertion" (Kaufmann 1932, 3212), that "the only subjects that can appear in language are concrete individual objects" (Kaufmann 1932, 3222), from which only numerical individual universals or general sortals can be

[14] On d.2) a clarification is in order. Although since January 1932 at least Kaufmann has had access to Carnap's preparatory manuscripts (1934), the expression *Bildungsgesetz* is already frequent in Kaufmann (1930, 95, 102, 123, 126, 128, 135-6, 143-4, 146, 149, 160, 167-9, 177) and in Kaufmann (1931, 286), and the expressions *Bildungsform* and *Umformung* had been used by Husserl, the former, in the field of morphology of meanings in (Husserl 1900-01, 152, 337, 658) and, the latter, in the field of syntax in (Husserl 1929, 311).

derived, thus declaring his linguistic physicalism or conceptual nominalism and at the same time agreeing with those who, like Gilles (1980), attribute a theory of tropes to Kaufmann;

f) the recognition that from these defined generalities, an iteration of the forms of reflection is certainly possible (Kaufmann 1932, 3217), but that in order not to fall into the vices of the branching type theory, the following principles must be adhered to:

f.1) do not confuse a concept with the objective aspect intended;

f.2) do not confuse determinations (internal or external properties) with mere names;

f.3) do not confuse nonsense with nonsense (Kaufmann 1932, 3238).

The 1930 Congress on the Foundations of Mathematics was behind us, as were the Kaufmann-Behmann *affaire* (in which the constructivist thesis had become far too radicalised), the manuscript on axiomatics and Gödel's announcement that if the transfinite axiom supplements the axiomatic system of classical mathematics, it is not formally provable. Moreover, if, as Gödel himself added (Hahn et al. 1931, 148), "in no formal system can it be stated with certainty that all the characteristics of the content are presentable", then it was precisely the Kaufmannian promise of recovering the semantic content of logical forms that was first and foremost disappointed. Re-reading Gödel more than ten years later, the programme to which Carnap was devoting himself in these years, of transforming not so much mathematics into syntax but philosophical reflections on mathematics into logical analyses of mathematical language (Carnap 1936a), did not fare any better. Nevertheless, Gödel (1944) saves one thing in Carnap (1931): the idea of a use of *all* that is not explained by tracing it back to individual cases[15].

From these comments, and from this brief overview, it would have been easy to predict the fate of the Carnap-Kaufmann confrontation and that of the difference between generalities would be sealed. In reality, this was not the case and, limited to the difference between generalities, this was, paradoxical-

[15] It seems to me that such a position can be found in Gödel (1944), who also quotes Langford (1927).

ly, truer for Carnap than for Kaufmann, who uses it a few more times in Kaufmann (1936, 39-40) and then loses it almost completely.

4.

Awodey, Carus (2009, 79) recount the genesis of Carnap (1934) as a drama in two acts: the first dating to January 1931, when Carnap drafted the 24-page *Attempt of Metalogic* after a sleepless night (Carnap 2022, 504-505), the second, dated to October 1932, when Carnap introduced the principle of tolerance. From (RC 110-04-07, 08 and 09) we know that the text had a first draft that began in July 1931 and ended in October 1932 and was read by Gödel, Behmann, Hempel, Kaufmann, Rand and Reach and a second draft from January to December 1933. We also know that the original title was *Metalogic*, later replaced by *Semantics* and finally, at Neurath's urging, by *Logical Syntax*. The 1932 index lists *Metalogic* as the title, *Logical Syntax of Language* as the subtitle, and *Semantics* as the title of the second part.

This periodisation is found point by point in the correspondence with Kaufmann, confirming that the latter was one of the main interlocutors of Carnap's Prague period.

On 1 January 1932 (KN 8109), Carnap writes to Kaufmann about a discussion that took place in those days with Hempel (30/12/1931-3/1/1932, Carnap 2022, 536) "particularly on the *Metalogic*, of which I have written a first draft". On 14/5/1932 (KN 8111), Kaufmann sends Carnap some comments on the first two chapters of the manuscript, including the *Descriptive Metalogic*, and informs him that Hempel will be giving a talk at the Reichenbach Circle on *Metalogic* in early summer. On 2/9/1932, Carnap wrote to Kaufmann (KN 8124) that the second part of his book had been typed, except for the last chapter on semantics and philosophy. On 11/10/1932, Kaufmann sends Carnap his remarks on *Semantics* (KN 8125), but these do not go beyond p. 336 of the manuscript, so, according to the 1932 index (RC 110-04-07), they do not yet include the last chapter, which was in fact written between September and November 1932. With regard to this last chapter, it is worth noting that the initial title "semantics and philosophy" is then replaced by "doctrine of science and semantics", divided into a first part (on the form of the propositions of the doctrine of science) and a second (the doctrine of science in the sphere of semantics), in which the occurrence of the Bolzanian-Husserlian locution of *Wissenschaftslehre*, instead of

Wissenschaftslogik, should not be overlooked[16] . On 30/11/1932, Kaufmann thanked Carnap for sending the last chapter of *Semantics* and informed him that he had lent the first part of the last paragraph to Rand (KN 8152). On 31/8/1933, Carnap informs Kaufmann of his hope that the book will be out by October (KN 8168). On 27/9/1933, Carnap writes to Kaufmann to thank him for his friendly intercession with the Rockefeller Foundation, with a view to his transfer to the USA (KN 8169) and encloses a list of his main publications, in which the title *Logical Syntax of Language* appears for the first time (KN 8170). The next day, Kaufmann replied on the one hand by advising him not to get his hopes up (the project, in fact, did not go ahead) and on the other by reassuring him that he would also plead his cause with the argument that "in a few months you will publish a book that will certainly become a standard work for fundamental research in logic and mathematics, and which is also of great interest to philosophers, that you have great pedagogical gifts and that your call to America would be of great inspiration to the philosophical circles of the universities in which you will be working" (KN 8172). On 14/10/1933, Carnap wrote to Kaufmann that the latest version of his book was very different from the one he had read, that he hoped it would be published by November, and, above all, he confessed that "the Brouwer-classical-mathematics question is not a question of correctness, but of decision, it consists in the choice between a poorer and a richer language" (KN 8175). On 13/1/1934, Carnap thanked Kaufmann for sending his review of FTL, which, in his view, offers a clear presentation of the Husserlian conception of logic, and confided in him that, despite the *Gutachten* of Russell, Boll, Rougier, Kaila, Neumann, Lewis, Huntington, Sheffer and Whitehead, the Rockefeller question seemed hopeless to him and that he would fall back on a visiting-professorship (KN 8178)[17]. On 22/6/1934, Carnap writes that the *Syntax* would finally come out after a few days and asks him if he would like to review it (KN 8192). On 6/7/1934,

[16] In Carnap (1934, 205) we read that the expression doctrine of science is not employed, "because it is more appropriate for the broader field of questions which, in addition to the logic of science, also includes the empirical investigation of scientific activity, and thus historical, sociological and, above all, psychological research". One may wonder how much weight Kaufmann's idea of *Wissenschaftslehre* already had in Kaufmann (1925). On this, I would refer to Masi (2022).

[17] On 1/3/1934 (KN 8182), Kaufmann wrote to Carnap of his regret at the negative response from the Rockefeller Foundations, motivated by the general rule of only funding scholars under 35.

Carnap sends a copy of the *Syntax* to Kaufmann (KN 8194). From 2/7/1934, the 8th *International Congress of Philosophy* was held in Prague: that congress was the premiere of *Syntax* for the philosophical public and the end of the Vienna Circle, two years before Schlick's death[18] .

The bulk of the comparison between Kaufmann and Carnap concerns topics from chapters I-IV of Carnap's edited version (1934). The main sources are the comments of 14/5/1932 (KN 8111) and those of 11/10/1932 (KN 8125). The main objections concern:

a) the definition of metalogics as language about language, i.e. the status of the language of what metalogics expresses itself about and the type of operations that allow one to speak about language;

b) the definition and application of 'formal'.

Kaufmann's main interest, on the other hand, concerns, as one might expect,

c) paragraph I. C: "remarks on the justification of the language model", i.e. what roughly corresponds to "remarks on the defined form of language".

In a) two of Kaufmann's already discussed convictions converge the neo-Aristotelian idea of realistic language and the clarifying task of philosophy. Suppose language is used properly only to refer to concrete individual objects. In that case, one can speak of language either as a physical set of inscriptions and sounds or to make explicit the references of language in proper or direct use. In the former case, one speaks of words that are just words (Husserl 1900-01, 10) or of the physical appearance of words (Husserl 1900-01, 47) and not of their linguistic or semantic function nor their correct syntactic combination. This is basically what descriptive metalogics would do by dealing with the "sensible material substratum of linguistic signs" (KN 8112). The two occurrences of 'word' are not distinguished according to common sense and its naive semantic faith in direct reference to the object, but according to two operations that Husserl had clearly distinguished: the physical perception of the sensible object 'inkblot' or 'sound' and the understanding of meaning. To be able to speak of the latter, I must make explicit or clarify what its reference consists of.

[18] At the Congress Husserl sent a paper entitled *The Present Task of Philosophy* (Husserl 1989, 184 ff.), which was to form the original core of the Vienna Lectures of the following year and thus of the *Crisis*.

If I explicate, I obtain evidence of distinction, i.e. I realise whether the sentence is syntactically correct or not; if I clarify, I obtain evidence of clarity, which concerns the formation of meaning, the claim to truth and possible confirmation. Both, explicitation and clarification, are at this time collected by Kaufmann under the common title of *Verdeutlichung*.

Hence, according to Kaufmann, the misunderstanding of 'formal' under b). The formal character of a term is not the determination of its figural moment without recourse to meaning but its type derived from the differences in meaning it can assume. What Kaufmann is thinking of is the reformulation of pure grammar since FTL's theory of nuclei, in which "the classification of different types of words is already a classification according to differences in meaning" (KN 8113). Evolution after Carnap (1934) certainly cannot be said to take up Kaufmann's criticism. Still, there is no doubt that it will respond to it, albeit with other resources: it will be the notion of translation, conquered by syntactic means, that will form the cornerstone of semantics. In 1932, however, Carnap's reply could not be starker: "I believe that the propositions of descriptive metalogics are really propositions about propositions, in which the object propositions are considered in a purely figurative way, without making any mention of the meaning of the figures. [...] 'Substantive' does not mean 'name for a thing', but something like 'word with a capital initial'" (KN 8121). Based on these two macro-biases, Kaufman then lists a series of analytical criticisms of the 1932 text (KN 8133-8138):

α) with regard to the introduction, Kaufmann objects that if "syntax is the system of formal rules for the formation of linguistic expressions", then it does not completely coincide with grammar since the latter would be broader, also encompassing inflexion; in this regard, he also recalls the example of 'Piroten korulieren elatisch', which, in his view, would be better explained by Husserlian pure grammar as a case of apparent grammaticality;

β) on the paragraph "linguistic signs and metalogical signs", Kaufmann believes that metalogical reformulation does not immediately make philosophical questions more precise but can serve as a heuristic tool to make thought distinct and consequential;

γ) on the paragraph "inferences in the language-model", i.e. on the first version of §10 on the transformation rules of (Carnap 1934, 25), Kaufmann asks what is meant when it is said that the antecedent and the consequent do not have a psychological, but a logical relationship, since they are not thought of together, but given together objectively (*objektiv mitgegeben*). Carnap's example significantly used a numerical universal quantifier, i.e. a peculiar form of restriction of generality, such that if all positions up to 5 are red, so are all positions up to 3. The problem for Kaufmann was the kind of self-evident understanding or method of verification of *objektive Mitgegebenheit*, and he also recalls FTL in this respect. In the same paragraph, Kaufmann points out the circularity of the statement according to which "the content of a formula is determined by the totality of the formulas, which can be derived from it"; but, Kaufmann notes, "does this not lead back to the already outdated confusion between individual and specific generality?" and would not an internal definition such as that of an analytic formula be better? In Carnap (1934, 38), the statement criticised by Kaufmann is replaced by the definition according to which the logical content of a proposition is the class of non-analytic propositions (of I) that follow from it, in which one should note both the limitation to the defined Language I and the use of consequence and not derivation, i.e. the relation that applies when from a proposition one can infer each individual case, but cannot infer the general proposition, i.e. when one takes an undecidable proposition (§36). It is difficult not to see the correlation between Kaufmann's criticism and the corrections introduced by Carnap.

δ) The only exception to Chapter I.C. is very significant. It concerns the statement that "only a theory can be demonstrated, whereas forms of language are conventional", which in some ways prefigures the principle of tolerance or, more precisely, Carnap's (1934, 42) observation that *Language I*, defined and intuitionist, is not the only possible or justifiable language. Still, they are part of a broader language, and above all that, both derive from a stipulation. In this regard, Kaufmann observed that a symbolism that could have different interpretations or that would confuse different meanings would be "false (in a figurative sense)" (KN 8134), i.e. ambiguous, perhaps still having in mind Carnap's (1931) non-bifurcability and monomorphy.

ε) Concerning the broadening of *Language I*, Kaufmann asks whether "the consequent positivist should not reject all mathematical concepts of existence and all existential definitions as meaningless, as long as they are not regarded merely as abbreviated formulations of constructions", thus identifying positivism, empiricism and constructivism.

ζ) On descriptions, i.e. what will be covered in Carnap (1934, 106 ff.) Kaufmann doubts whether it is possible to give an unambiguous syntactic description of the general form of the number.

η) Concerning the analysis of the difference between implication and derivability, which in Carnap (1934, 192 ff.) corresponds to the difference between implication and consequence relation, based on the assumption that while consequentiality is a relation between propositions, the implication is a relation between what is designated by the propositions, Kaufmann objects that it is not possible to distinguish between relations of propositions, relations of states of affairs and relations of truths. The point of the objection is to reiterate once again that the only organ of reflection on language is clarification and that this does not disassociate the proposition from its meaning. Kaufmann, however, disputes the necessity of distinguishing implication and derivability, i.e. formal and strict implication. At the end of this remark, Kaufmann adds an interesting observation to understand better his comparative reading of Carnap (1934) and Husserl (1929). "In this respect, Husserl (1929, 93 ff.) speaks of a change of gaze and correspondence. According to the view, which I have tried to justify better over the last few months, it is not really a matter of bilaterality [scil.: between proposition and state of affairs, between apophany and ontology]. But a serious problem remains here' (KN 8134-8135).

θ) On the paragraph in IV A about the difference between extensional and intentional formulas, Kaufmann points out that he no longer considers the Russellian justification of the extensionality thesis to be valid and that he is moving towards a position akin to that of the Brentanian theory of judgement, without however specifying it more precisely (KN 8135). If this is how he intended to understand not only the general intentional character of judgement but, more specifically, the thesis of the primitiveness of the existential form, as expressed in Brentano (1874) and Brentano (1930) and later specified in Chisholm (1982), this would be a significant departure from Husserl, which would restrict the choice of language even further. It should also be noted that extensionality in Carnap (1934) is no longer a thesis as it was in Carnap (1928a), but a hypothesis, and that while it is reiterated in Carnap (1934, 202) that non-formal logic is a *contradictio in adjecto* and that logic is syntax, it is emphasised that syntactic analyses are indifferent to the intensional or extensional interpretation of language. On the contrary, starting with Frege, more recent systems of logic would go beyond the limitation of the analysis of concepts only on their extension to the point of giving the impression of having eliminated extension in favour of intentionality, as, for example, in the arguments of no-class theory (Carnap 1934, 202).

ι) Kaufmann writes a long commentary on a paragraph in IV.B of the 1932 version, which no longer appears in 1934 and which was on the subject of probability[19] . This is a not insignificant circumstance since, in the 1940s, this was to be the central theme of the American confrontation between Kaufmann and Carnap. In this regard, Kaufmann first notes that the sphere of probability is not only that of relative frequencies and statistics but coincides with that of the whole of experience. Therefore, "when determining the measure of the range of a die, one involves the whole – *cum grano salis* – physically relevant experience" (KN 8135). Kaufmann also proposes, citing Poincaré's and Smoluchowski's precedents, to replace Mises' principle of irregularity and equiprobability with a formulation of the "causal independence of the phenomenal series P_1 (which is under scrutiny) from variations in the phenomena of other series P_1, regardless of whether the individual phenomena of the series P_1 are dependent on the individual phenomena of the series P_1"(KN 8136). However, such an independence thesis would not be a law of causality at all.
κ) The last remark concerns the paragraph in IV.D (relation theory and axiomatics) on internal and external isomorphy. Kaufmann advises Carnap to make more explicit the position, gained through syntactic descriptions, on the possibility of non-denumerable infinite sets.

At first glance, one notices two things from Kaufmann's general considerations and list of specific objections: that the recurring criticism of Metalogics/Semantics through *Verdeutlichungsthematik* closely resembles the criticism of Wittgenstein and the *Right Wing* of the Circle, and that the difference between specific and individual generality only appears in γ).

Regarding the first impression, it should be noted that Kaufmann did nothing to dispel it, rather perhaps he used it to give more weight to his criticism, as can be seen for instance from (KN 8130). Moreover, Ingarden (1934), at the Prague Congress already mentioned, also made similar objections. If two clues almost make a proof, it could be deduced that the phenomenologists, and their pure grammar, were attested to Wittgensteinian-like posi-

[19] In Carnap (1934, 244), probability is discussed only in physicalist language and without examining its definition.

tions. Now, if it is true that the dry land from which Carnap (1934, 18) departed to the 'boundless ocean of limitless possibilities' was the dry land of Tractarian molecular language, then, by extension, Pure Syntax beat Kaufmann and his phenomenological-realist language into the breach. If this were the case, it would indeed seem strange that Bar-Hillel (1957, 1970), who was not so badly acquainted with Carnap's thought, would claim that Carnap (1934) betrayed a strong influence of Husserl.

It is worth noting, then, in addition to Kaufmann's distancing himself from the pictorial conception of language (which I have already mentioned), the difference between *Erläuterung* and *Verdeutlichung*[20] . In Wittgenstein (1922), 3.263, we read that "The meanings of primitive signs can be explained by elucidations. Elucidations are propositions which contain primitive signs. They can, therefore, only be understood when the meanings of these signs are already known" and in 4.112 that "A philosophical work consists essentially of elucidations". The definitions of *Verdeutlichung* found in (Husserl 1929, 63, 71, 193, 197), on the other hand, call into question the difference between distinction and clarity, i.e. between pure grammar and epistemology, the role of reflection, the justification of the extensional interpretation of language, and extend from here towards the broader theme of explication that will be central in Husserl (1939)[21] .

In short, *I do not believe that Kaufmann's phenomenological realist language is the* terra firma *or the prison from which Carnap* (1924; cf. Awodey-Carus 2009) *frees himself, but the well-defined thing language of ordinary experience, which constitutes the indispensable prerequisite for any liberalisation of the empirical criterion of significance* (cf. Klev 2018). Its defi-

[20] In order to get a more complete picture, it would also be necessary to highlight the difference between Wittgenstein's conception of tautology (1922) and that contained in Becker's appendix to Husserl (1929), and from there the different transcendental arguments that allow Wittgenstein to say that tautology is *sinnlos*, i.e. without a grip on the world or on what happens to be, and to Husserl that the check on non-contradiction and consequentiality does not make it possible to establish whether a proposition is true or not.

[21] This is not the place to go into further detail, but suffice it to say that the question (not only logical-philosophical, but also civil and, I would say, political) of clarification is so decisive for Kaufmann, especially in the phase following this confrontation and in the American phase, that he worked in his later years on a book, which has remained unpublished, entitled *The Pursuit of Clarity*, in which these arguments are arranged in a peculiar phenomenological-pragmatist framework.

nition makes it something extremely different from protocol language as well as from observational language and any formulation of a language made up of occasion statement: it is not phenomenalistic-solipsistic like the former, it does not contain only observational terms like the latter, nor is it configured as a direct, reactive response to external stimulus like the third. Its distinctive character lies precisely in which and how many non-thing terms it admits, according to which criterion it does so, and which control procedure it establishes. This is why the difference between generalities is indispensable.

And so, we come to the second impression: that the difference between generalities was somehow marginal in the *Remarks*. Kaufmann had indeed premised that he would concentrate on criticism and not praise; he forgot to admit, however, that the model language, i.e. *Language I* (henceforth LI), was the formalisation, albeit not always faithful, of his neo-Aristotelian language[22] . The only clarification in this regard to be found in Kaufmann (KN 8114) is: "my position coincides completely with that of his model language, in particular with his distancing himself from intuitionism; this also applies to Law of Excluded Middle (henceforth LEM). I do not believe that my perspective is definable as intuitionist (in Brouwer's direction)".

Carnap (1934) uses Kaufmann in four places, the first two of which are strategic: in § 16 on intuitionism, § 17 on the principle of tolerance, § 38 on the elimination of classes and § 43 on the admissibility of undefined concepts. It is, however, in the construction of the definite, closed, and constructive language of L I that Kaufmann's function is decisive. Without Kaufmann and the difference between generalities, it would not have been possible to construct L I, the importance of which is difficult to underestimate: it is based on it, in fact, that one can distinguish between consequentiality and derivability and thus unambiguously establish the acceptability of non-predicative definitions and thus finally accept the complete undecidability of an AS. However, two things must be noted: firstly, Carnap's use of specific generality is incorrect, as will become clear from the analysis of the example in Carnap (1934, 44); secondly, the notion of individual generality generates multiple formula-

[22] I recall that at the end of his review of Kaufmann (1930), Carnap hoped that just as Russell (1910-1925) had derived a formal construction of Russell's (1903) logical-mathematical system, and as Heyting had done the same with Brouwer, so too would it be done with Kaufmann (1930).

tions of restricted generalities, including that of the numerical operator K (Carnap 1934, 21).

L I is a language that speaks of any objects within a sphere and can designate these objects with proper names or with systematic positional coordinates, i.e. through symbols that let us know their location in the system and their mutual position (Carnap 1934, 11). This is a definite language, albeit not in the strict sense, because it admits confirmation procedures consisting of a finite number of steps but does not exclude propositions that cannot be conclusively proved or disproved. In L I, in addition to ordinary sensible experience, the elementary arithmetic of natural numbers and the part of physics that uses natural numbers to designate the four space-time coordinates are also expressible. In addition to names, or variables, and positions, or numerical constants, L I admits predicates (descriptive or logical), understood as the names of the properties of positions, functors (descriptive or logical), i.e. numerical expressions of predicates, and a logical vocabulary, which reduces connectives to negation and conjunction only, admits only existential quantification and the definite numerical operator, through which a definite description of a given interval or the number of elements of a finite set is accomplished[23]. The operator K, Carnap (1934, 21) adds, differs from the Russellian definite descriptions in that it is never empty (even if this means that it is not committed to its existential quantification (Carnap 1934, §38a)) and is not equivocal, so that in L I not only is the ramified theory of types excluded (which is not indispensable for L II either, Carnap (1934, 76-77), but Ramsey's simple version is not used either. In fact, if we assume that 'Ma (a, b)' stands for 'a is greater than b', then

$$(Kx)9(Ma(x,7))$$

means 'the number smaller than 9 that is greater than 7', i.e. 8.
The universal and existential operators are only needed in I under limitation, and in order to achieve unlimited universality, free variables are used that are not replaceable by any element of a domain (though still limited), but by any element, so that

[23] The addition made by Carnap (1934, 11) that the language of proper names is more original than that of co-ordinates has no constructive reason, nor is it a concession to phenomenological realist language, but rather obeys a so to speak evolutionary conception of language, from a phase in which object identification is more concrete and less unambiguous, to one in which it is less concrete and more unambiguous.

$$(x)3\,(red\ (x)) \equiv red(0) \wedge red(1) \wedge red(2) \wedge red(3)$$

i.e. 'all positions up to 3 are red',

$$(\exists x)3\,(red\ (x)) \equiv red(0) \vee red(1) \vee red(2) \vee red(3)$$

i.e. 'at least one of the positions up to 3 is red', and especially

$$sum(x, y) = sum(y, x)$$

which is obviously not to be interpreted as 'for each *x* and each *y*, the sum of the first and the second is equal to the sum of the second and the first' (since the universal operator is not placed before it), but as 'for any two numbers, ...'. Again, as with the operator K, the free variable has an unambiguous and determined value (Carnap 1934, 20).
It follows from this treatment of unlimited universality that in L I, it can only be positively asserted but not denied, as is the case in intuitionist mathematical language. However, unlike in Carnap (1934, 42), with regard to I this does not mean that

$$P(x)$$

and

$$\neg P(x)$$

are expressible because they are to be understood as 'all *x* are *P*' and 'all *x* are not *P*' respectively, but because, containing free variables, they are to be understood as 'for any *x* it is *P*' and 'for any *x* it is not *P*', whereas they are not expressible

$$(x)(P(x)), \neg(x)(P(x)), (x)\neg(P(x))$$

i.e. 'all *x*', 'not all *x*' and 'all *x* are not *P*'. However, this difference blurs because Carnap intends to present L I as a language that is at least partially acceptable to intuitionists (among whom, besides Poincaré, Brouwer and Heyting, Weyl and Becker are mentioned, and, on similar positions, Wittgenstein and Kaufmann), what is most relevant is that the introduction of the restricted quantifiers is not justified intuitionistically. The exclusion of unlimited quantifiers applied to the problem of indirect proof (which had been Kaufmann's worry) allows the LEM to remain valid, as already argued in Kaufmann (1930) and reiterated in the cited letter of 14/5/1932 (KN 8114). In L I, for example, one can substitute

$$(x)(P(x))$$

which it is impossible to reduce to absurdum with

$$(P(x))$$

but you cannot do the same with

$$\neg(x)(P(x))$$

nor with

$$(\exists x)(\neg(P(x))$$

without thus touching the LEM[24] .

At this point, Carnap (1932, 43-44) adds that the two different ways of expressing generality, by means of free variables and by means of limited universal operators, correspond to two types of generalities: specific generality and individual generality. And he makes three clarifications: that it is left open whether Kaufmann's criticism of PM and set theory is justified or not, that L I could be the realisation of part of Kaufmann's ideas, that Kaufmann, like Wittgenstein, does not accept *unlimitedly general synthetic* propositions, i.e. propositions that are neither analytical (i.e. specific general) nor limited (i.e. individual general), because they cannot be completely verified.

The most important clarification is obviously the third. It can be explained through the three examples of generalities that Carnap (1934, 44) gives:

1. 'all the pieces of iron that are on this board are round',

2a. 'all pieces of iron are pieces of metal',

2b. 'all iron pieces are magnetisable'.

In 1, a limited universal operator is required and is synthetic, 2a and 2b express unlimited generality and can be formalised by means of free variables, but 2a is analytical, while 2b is a hypothesis and is synthetic, like the laws of nature.

[24] In truth, taken (P(x)), one could derive in I
(P(x)∨¬(P(x)) LEM
and thus
¬(P(x))≡(∃x)n¬(P(x))
that is, if and only if, for any *x*, it is not the case that it is *P*, then some *x*, up to *n* (which is countable), is not the case that it is *P*.

According to Carnap, the first corresponds to an individual generality, the second to a specific generality, while the third, being neither, is inexpressible in Kaufmann's language.
One cannot help but notice, however, that 2a is not a good example of specific generality at all, or at least it is a very tendentious example, since if it corresponds to an analytic proposition and an analytic proposition is defined as L-true, i.e. as a proposition expressing equality of content or synonymy, the introduction of specific generality is completely unnecessary[25]. Admittedly, in Carnap (1934, 39), Dubislav (1926) and the notion of the analytic relative to a specific language are cited, but no reference is made to this in examining 2a. For the reasons I have already given and will now partly repeat, it would nevertheless have been appropriate to explain specific generalities as relatively analytic. Only much later, in the posthumous Carnap (1971) and quoting Delius (1963), will Carnap recognise that an analytical continuum can be delineated through specific generalities.

But let us return to the examples. 2a is general specific because it expresses the subordination of a species (iron) to a genus (metal); the same relationship applies between red and colour or between musical note and sound. A proposition such as 'any colour is extended', or 'any chromatic quality is extended', or 'any sound has a pitch and intensity' and the like (including Schlick's black beast: 'no body can be entirely green and red'), are all general specific, but do not express subordination at all. What they all have in common, however, is that they express an incompatibility between the subsistence of any individual in a given field and the non-existence of at least one property. Moreover, their generality is not distributive but collective, and this is decisive for their method of confirmation, which, for this very reason, cannot be complete. Depending on whether the restriction on the sensible field is made explicit, a statement such as "any sound has a pitch and an intensity" could be formally translated as

$$\neg(S(x)) \vee \big(\big(A(x)\big) \wedge (I(x))\big)$$

if I do not make the field explicit and use free variables or with

[25] Let us, of course, leave aside whether this definition of the analytic is sustainable and whether after Quine's objections the introduction of the specific generality does not serve, at least in part, to restore the meaning of the analytic.

$$(\exists x)c(\neg(S(x)) \vee \big(A(x)\big) \wedge (I(x)))$$

by explicating the field with c and using a restricted existential operator or with

$$(\forall x)c(\neg(S(x)) \vee \big(A(x)\big) \wedge (I(x)))$$

using this time a restricted universal operator and only allowing for a collective reading.

As for 2b, however, it is worth recalling that in the case of the so-called laws of nature, Kaufmann spoke of similarities or morphological connections that can be expressed by means of individual generalities, or it would be better to say, restoring the initial wording, collective generalities, because these too do not admit of distributive interpretation or definitive confirmation[26]. If this is the case, however, 2b is expressible in Kaufmann's language but does not represent an unlimitedly general synthetic proposition. It is still worth noting that the classification of the three examples into analytic (2a) and synthetic (1 and 2b) would no longer be respected since 1 and 2b would remain synthetic, so to speak, while 2a would admit a dual interpretation: as analytic, in the use of the free variable, or as the statement of a law, and synthetic, in the use of restricted operators. Despite the infidelity of the examples, therefore, Carnap undoubtedly contributes to making the formulation of specific generality more formally rigorous and placing it as a hinge between reality or positional (synthetic) propositions and property and non-positional (analytic) ones.

The other references to Kaufmann are his critique of the Russellian concept of class, which Carnap (1934, 101) considers being rather directed at the arguments that Russell uses to support the adoption of that concept, and his rejection of indefinite symbols, i.e. those in whose chain of definitions at least one unlimited operator is needed, since there is no method of decision for them (Carnap 1934, 114), and of impredicative terms, as well as any apparent

[26] Still on 2b, however, one could argue for a particular application of the Husserlian distinction between law and necessity, limited to the physical realm. On the basis of this distinction, which, however, Kaufmann never uses, I can establish (hypothetically or conventionally) a law, which is analytically limited to a set of more or less broad assumptions, and which expresses an invariable correlation in a certain field, and derive from it a series of applications each of which would express the necessary application of such a correlation. A special case of applications would also be probabilistic assumptions.

variables (Carnap 1934, 117). Furthermore, the reformulation of the principle of tolerance that appears here, i.e. "in what way will we construct a particular language" (Carnap 1934, 117), is very similar to that contained in the remarks to Kaufmann (1932) and to that found in Weyl (1918) in the transition from restricted to extended language.

5.

We have thus finally reached the end of our story. On 7/6/1935, Kaufmann, who is engaged in the last revisions of Kaufmann (1936), writes to Carnap about how convincing the introduction of indirect controllability was for him in Carnap (1935) and how it gives him hope for a reformulation of physicalism and, more generally for the completion of the project of rational reconstruction, which for him coincides with that of the clarification of "what is really supposed to think" (KN 8210). Kaufmann confesses that he hoped that this extension of controllability would convince Carnap that the problems of the theory of science cannot be reduced to linguistic questions and, in particular, that one can also depart from the principle of tolerance, "from this conventionalist principle that certainly in many cases acts as an antidote against the metaphysics of concepts, when one emphasises the 'freedom of definition', [but] this freedom fades as soon as one is dealing with the rational reconstruction of the meaning attached to a term and one plunge into methodological disputes in the various sciences". On 24/5/1937, when Carnap is already in Chicago, Kaufmann writes that he has read Carnap (1936-37) with great pleasure and that he finds in it the confirmation of the hope of two years earlier (KN 8225).

Indeed, Carnap (1936-37) extends and liberalises the empirical concept of significance, distinguishing between definition and reduction, between controllability and confirmability, each of which has degrees, and defining a scale of languages on this basis. The first of these languages is the molecular language (L_0), which would allow complete controllability and confirmability. L_0 is a thing language, i.e. 'that language which we use in everyday life by talking about the perceptible things around us' (Carnap 1936-37, 208). In such a language, one could formulate an assertion as "on 6 May 1935, at 4 p.m., there is a round black table in my room".

As trivial as this statement is, it is impossible to translate it into a finite series of statements about possible perceptions. Such an observation was found in Kaufmann (1930) in the form of the principle of inexhaustibility or transcendence, which, together with that of accessibility, formed the basis of his peculiar form of phenomenology. It is not for this reason, however, that Carnap (1936-37, 232) mentions Kaufmann, but to place him in a sort of period picture of the Circle's roaring years, when Schlick, Wittgenstein, Carnap himself and Kaufmann argued that L_0 was "*the* language, that is, the only possible language". Of Kaufmann (1930), we are also reminded of his rejection of non-restricted universals, except for a priori universals. This clarification did not appear in Carnap (1934), where instead, he merely referred to them as analytical.

In that letter of May 1937, Kaufmann informs Carnap that he will attend the 9th International Congress of Philosophy in Paris with Kaufmann (1937) in the summer. In one of the three papers at the same Congress, Carnap (1936) will return to thing language. Kaufmann (1937), on the other hand, returns to the ambiguity of the notion of formal and formalisation that he had already discussed, as well as his critique of the conception of linguistic signs as figural moments or sign substrates. The focus of the essay is the analysis of the proposition "what applies to the general, also applies to the particular", and the key to its interpretation is found in the "implied possibility of the knowledge of a general" (Kaufmann 1937, 132). Three arguments, however, are of greatest interest:

1) the correlation between deduction and specific or conceptual generality, as he now defines it, on the one hand, and induction and individual or numerical generality on the other (Kaufmann 1937, 128);
2) a renewed focus on the relationship between concept and operation and thus the beginning of the elaboration of his system of rules of scientific procedure (Kaufmann 1937, 131-133);
3) a semi-formal clarification of the meaning of incompatibility.

Incompatibility is understood, in fact, as a logical relation invariant to every possible meaning of concepts and propositions but not independent or indifferent to these meanings. It is precisely for this reason that the principle of contradiction, as well as LEM, are two possible expressions of two different types of incompatibility and not the logical laws from which the incompatibility derives or of which the incompatibility is an application. It is no coincidence that Kaufmann (1937, 133) gives the example of the incompatibility between 'D is red' and 'D is - at the same point in time and in the same position - blue'.

What is incompatible are two propositions referring to the same object, designated by name or position: the two propositions cannot both be true. But this does not mean that they cannot both be false. That 'D is red' is incompatible with 'D is blue' does not mean that 'D is red if and only if D is not blue', but that 'If D is red then it is not blue'[27] .

*

In 1938, Kaufmann also emigrated to the United States and the following decade saw another phase of the confrontation with Carnap centred on the concepts of truth, probability, and scientific procedure. The difference between generalities seems to disappear from the radar, only recovered later in the guise of Dewey's (1939) distinction between generic and universal propositions.

What Carnap and Kaufmann witnessed between 1928 and 1934 was the only case of a real and living confrontation between phenomenology and neo-empiricism and one of the very rare examples in which a concept was formed in exchange and mutual correction. The Rhine and the Danube, which had briefly mixed their waters, would diverge again, reducing their scope and perhaps that of the very idea of empiricism.

.

REFERENCES

Awodey, S., Carus, A. (2001), Carnap, completeness, and categoricity: the Gabelbarkeitssatz of 1928, *Erkenntnis*, 54, 2, 145-172.

Awodey, S. Carus, A. (2009), From Wittgenstein's Prison to the Boundless Ocean: Carnap's Dream of Logical Syntax, in Wagner (2009), 79-108.

Bar-Hillel, J. (1957), Husserl's conception of a purely logical grammar, *Philosophy and phenomenological research,* 17, 3, 362-369.

Becker, O. (1924), Beiträge zur phänomenologischen Begründung der Geometrie und ihrer physikalischen Anwendungen, *Jahrbuch für Philosophie und phänomenologischen Forschung*, 6, 385-560.

Behmann, H. (1930), Zur Frage der Konstruktivität von Beweisen, Unpublished typescript, Wissenschaftshistorische Sammlung, ETH, Zurich, Hs 974: 18.

[27] In fact, assuming 'D is blue' and 'D is red', while saying "the truth of one corresponds to the falsity of the other" is equivalent to $(Bd \land \neg Rd) \lor (\neg Bd \land Rd) \equiv (Bd \leftrightarrow \neg Rd)$, saying 'they cannot both be true' is equivalent to $(\neg Bd \lor \neg Rd) \equiv (Bd \rightarrow \neg Rd)$.

Brandom, R. (2008), Between Saying and Doing, OUP, Oxford.

Brentano, F. (1930), Wahrheit und Evidenz, Meiner, Leipzig.

Bolzano, B. (1837), Wissenschaftslehre, hrsg. von F. Kambartel, Meiner, Hamburg.

Boolos, G. (1984), To Be is to Be a Value of a Variable (or to Be Some Values of Some Variables), *Journal of Philosophy*, 81, 430-449.

Boolos, G. (1985), Nominalist Platonism, *Philosophical Review*, 94, 327-44.

Carlson, G.N. (1977), Reference to Kinds in English, Ph.D. dissertation, University of Massachusetts, Amherst.

Carnap, R. (1927), Eigentliche und Uneigentliche Begriffe, *Symposion*, I, 355-374.

Carnap, R. (1928a), Der logische Aufbau der Welt, Penary, Berlin.

Carnap, R. (1928b), Untersuchungen zur allgemeinen Axiomatik, ed. T. Bonk and J. Mosterin, Wissenschaftliche Buchgesellschaft, Darmstadt, 2000.

Carnap, R. (1929), Abriss der Logistik, Springer, Wien.

Carnap, R. (1930), Bericht über Untersuchungen zur allgemeinen Axiomatik, *Erkenntnis*, 1, 303-7.

Carnap, R. (1931), Die logizistische Grundlegung der Mathematik, *Erkenntnis*, II, 91-105.

Carnap, R. (1934), Logische Syntax der Sprache, Springer, Wien.

Carnap, R. (1935), Les concepts psychologiques et les concepts physiques sont-ils foncièrement différents?, *Revue de Synthèse*, 10, pp. 43-53.

Carnap, R. (1936), Wahrheit und Bewährung, in Aa. Vv., Actes du Congrès International de Philosophie Scientifique, IV, Hermann, Paris, 18-23.

Carnap, R. (1936-37), Testability and Meaning, *Philosophy of Science*, III, 419-71; IV, 1-40.

Carnap, R. (1942), Introduction to Semantics, HUP, Cambridge.

Carnap, R. (1967), Logical Structure of the World, UCP, Berkeley.

Carnap, R. (1971), Basic System of Inductive Logic, in R. Carnap, R. C. Jeffrey, Studies in Inductive Logic and Probability, I, UCP, Berkeley-Los Angeles.

Carnap, R. (2022), Tagebücher, Band 2: 1920-1935, hrsg. von Ch. Damböck, Meiner, Hamburg.

Carus, A. (2010), Carnap and the Twentieth-Century Thought. Explanation as Enlightenment, CUP, Cambridge.

Chisholm, R. (1982), Brentano's Theory of Judgement, in Brentano and Meinong Studies, Rodopi, Amsterdam, 17-36.

Couturat, L. (1905), Principes des mathématiques, Paris, Alcan.

Dahms, H.-J. (1997), F. Kaufmann und der Physikalismus, in Stadler (1997), 97-114.

Delius (1963), Untersuchungen Zur Problematik der Sogenannten Synthetischen Sätze Apriori, Vandenhoeck & Ruprecht, Gottingen.

Dewey, J. (1939), Logic, Theory of Inquiry, in Id., Later Works, 12, ed. by J. A. Boydstone, Southern Illinois UP, Cardondale-Edwardsville.

Dummett, M. (1973), Frege. Philosophy of Language, Duckworth London.

Dummett, M. (1991), The Logical Basis of Metaphysics, HUP, Cambridge.

Dummett, M. (1996), The Seas of Language, OUP, New York.

Dubislav, W. (1926), Über die sogenannten analytischen und synthetischen Urteile, Weiss, Berlin.

Dubislav, W. (1931), Bolzano als Vorläufer der mathematischen Logik, *Philosophisches Jahrbuch der Gorres-Gesellschaft*, 44, 448-456.

Fraenkel, A. (1928), Einleitung in die Mengenlehre, Springer, Berlin.

Frascolla, P. (1994), Wittgenstein's Philosophy of Mathematics, Routledge, London.

Geiger, M. (1924), Systematische Axiomatik der Euklidischen Geometrie, Filser, Augsburg.

Gödel, K. (1931), Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I, *Monatshefte für Mathematik und Physik,* 38, 173-198.

Gödel, K. (1944), Russell's Mathematical Logic, in P. A. Schilpp (ed.), The Philosophy of Bertrand Russell, Northwestern University Press, Evanston, 123-153.

Golbfarb, W. (2009), Carnap's Syntax Programme and the Philosophy of Mathematics, in Wagner (2009), 109-120.

Hahn, H. *et al.* (1931), Diskussion zur Grundlegung der Mathematik (H. Hahn, R. Carnap, J. Neumann, H. Scholz, A. Heyting, K. Gödel, K. Reidemeister) *Erkenntnis*, 10, 135-149.

Husserl, E. (1891), Philosophie der Arithmetik, Hua 12, hrsg. von L. Eley, Nijhoff, Den Haag 1970, 5-283.

Husserl, E. (1900), Logische Untersuchungen I, Hua 18, hrsg. von E. Holenstein, Nijhoff, Den Haag 1975.

Husserl, E. (1900-01), Logische Untersuchungen II, Hua 19/1, hrsg. von U. Panzer, Kluwer, Dordrecht 1984.

Husserl, E. (1901), Das Imaginäre in der Mathematik, in Hua 12, 430-451.

Husserl, E. (1913), Ideen zu einer reinen Phänomenologie und phänomenologischen Philosophie, I, Hua 3/1, hrsg. von K. Schuhmann, Nijhoff, Den Haag 1976.

Husserl, E. (1929), Formale und transzendentale Logik, Hua 17, hrsg. von P. Janssen, Nijhoff, Den Haag 1974.

Husserl, E. (1939), Erfahrung und Urteil, Meiner, Hamburg 1999.

Husserl, E. (1979), Aufsätze und Rezensionen (1890-1910), Hua 22, hrsg. von B. Rang, Nijhoff, Den Haag.

Husserl, E. (1989), Aufsätze und Vortrage (1922-1937), Hua 27, hrsg. von Th. Nenon und H. R. Sepp, Kluwer, Dordrecht.

Husserl, E. (1994), Briefwechsel, Hua, Mat. 3/7, hrsg. von E. und K. Schuhmann, Springer, Dordrecht.

Husserl, E. (2001), Logik. Vorlesung 1896, Hua, Mat. 1, hrsg. von E. Schuhmann, Springer, Dordrecht.

Husserl, E. (2003), Alte und neue Logik. Vorlesung 1908-1909, Hua, Mat. 6, hrsg. von E. Schuhmann, Springer, Dordrecht.

Ingarden (1934), Der Logistische Versuch einer Neugestaltung der Philosophie: Eine Kritische Bemerkung, Aa. Vv., Actes du huitième Congrès internationale de Philosophie à Prague, Prag, 203-08.

Kaufmann, F. (1921), Logik und Rechtswissenschaft, Tübingen, Mohr.

Kaufmann, F. (1925), Logik und Wirtschaft. Eine Untersuchung über die Grundlagen der ökonomischen Theorie, *Archiv für Sozialwissenschaft und Sozialpolitik*, 54, 614-565.

Kaufmann, F. (1930), Das Unendliche in der Mathematik und seine Ausschaltung, Leipzig and Vienna.

Kaufmann, F. (1931), Bemerkungen zum Grundlagenstreit in Logik und Mathematik, *Erkenntnis* 2, 262-290.

Kaufmann, F. (1932), Logische Prinzipienfragen in der mathematischen Grundlagenforschung (KN 3204-3254).

Kaufmann, F. (1936), Methodenlehre der Sozialwissenschaften, Springer, Vienna.

Kaufmann, F. (1937), Über den Begriff des Formalen in Logik und Mathematik, in Aa. Vv., Travaux du 9.éme Congres International de Philosophie, 1937, 6, 128-135.

Kaufmann, F. (1944), Methodology of the Social Sciences, OUP, London-New York.

Kaufmann, F. (1978), *The Infinite in Mathematics. Logico-mathematical Writings*, ed. by B. McGuinness, Reidel, Dordrecht.

Klev, A. (2018), Carnap's Turn to the Thing Language, *Philosophia Scientiae*, 22-3, 179-198.

Kriegel, U. (2004), Trope theory and the metaphysics of appearances, *American Philosophical Quarterly*, 41, 1, 5-20.

Langford, C.H. (1927), An analysis of some general propositions, *Bulletin of American Mathematical Society,* 33, 6, 1927, 666-672.

Linnebo, Ø. (2022a), Generality Explained, *Journal of Philosophy*, 119, 7, 349-379.

Linnebo, Ø. (2002b), Plural Quantification, in Stanford Encyclopaedia of Philosophy.

London, F. (1923), Die notwendige Bedingung der Möglichkeit einer deduktiven Theorie, *Jahrbuch für Philosophie und Phänomenologische Forschung*, 6, 335-384.

Mancosu, P. (2010), On the Constructivity of the Proof. A Debate among Behmann, Bernays, Godel and Kaufmann, in Id., The Adventure of Reason, OUP, New York, 199-216.

Marion, M. (1998), Wittgenstein, Finitism, and the Foundations of Mathematics, OUP, Oxford 2008.

Masi, F. (2022), Specialist in Generality. F. Kaufmann e l'interpretazione empiristico-critica della metodologia weberiana, in G. Morrone, C. Russo Krauss, D. Spinosa, R. Visone (eds.), Max Weber Constellation, Fedoa Press, Naples, 267-303.

Mormann (1991), Husserl's Philosophy of Science and the Semantic Approach, *Philosophy of Science*, 58, 1, 61-83.

Nagel, Th. (1978), Introduction in Kaufmann (1978), IX-XV.

Parrini, P. (1977), Presentazione a Russell-Whitehead, Introduzioni ai Principia Mathematica, La Nuova Italia, Florence, VII-LVII.

Parrini, P. (1980), Una filosofia senza dogmi: materiali per un bilancio dell'empirismo contemporaneo, Il Mulino, Bologna.

Parrini, P. (1998), Knowledge and Reality. An Essay in Positive Philosophy, Springer, Dordrecht.

Parrini, P. (2022), Empirismo logico e fenomenologia. Uno snodo fondamentale della filosofia del Novecento, in F. Masi, R. Melisi, F. Seller (eds.), Tra experientia ed experimentum, Mimesis, Milano-Udine, 103-115.

Popper, K.R. (1934), Logik der Forschung, Springer, Wien.

Proust, J. (1986), Questions de forme. Logique et proposition analytique de Kant à Carnap, Fayard, Paris.

Quine, W.V.O. (1947), On Universals, *The Journal of Symbolic Logic*, 12, 3, 74-84.

Quine, W.V.O. (1948), On What There is, *Review of Metaphysics*, 2, 5, 21-38.

Quine, W.V.O. (1987), Quiddities, Belknap, Cambrdige.

Ramsey, F.P. (1926), The Foundations of Mathematics, in Id., The Foundations of Mathematics and other Logical Essays, Routledge & Kegan, London, 1-61.

Russell, B. (1903), Principles of Mathematics, CUP, Cambridge.

Russell, B. (1908), Mathematical Logic as Based on a Theory of Types, *American Journal of Mathematics*, 30, 222-262.

Russell, B. (1910), Principia Mathematica (with Whitehead), 1. ed., CUP, Cambridge.

Russell, B. (1925), Principia Mathematica (with Whitehead), 2. ed, CUP, Cambridge.

Schlick, M. (1918), Allgemeine Erkenntnislehre, GA, I/1, hrsg. H. J. Wendel und F. O. Engler, Springer, Wien-New York.

Scholz, H. (1931), Abriss der Geschichte der Logik, Alber, Frankfurt/München.

Uebel, T. (2009), Carnap's Logical Syntax in the Context of the Vienna Circle, in Wagner (2009), 53-78.

Veblen, O. (1904), A System of Axioms for Geometry, *Transactions of the American Mathematical Society*, 5, 343-384.

Venn, J. (1894), Symbolic Logic, MacMillian & C., London.

Wagner, R. (2009), Carnap's Logical Syntax of Language, Palgrave, New York.

Waismann, F. (1979), Wittgenstein and the Vienna Circle, Rowman & Littlefield, Lanham.

Weyl, H. (1918), Das Kontinuum, Veit & C., Leipzig.

Weyl, H. (1921), Über die neue Grundlagenkrise der Mathematik, *Mathematische Zeitschrift*, 10, 1-2, 39-79.

Williamson, Th. (2003), Everything, *Philosophical Perspectives,* 17, 1, 415-465.

Wittgenstein, L. (1922), Tractatus logico-philosophicus, Routledge & Kegan, London 1960.