Contradictions in Motion: Why They're not Needed and Why They Wouldn't Help

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ABSTRACT

In this paper we discuss Priest's account of change and motion, contrasting it with its more orthodox rival, the Russellian account. The paper is divided in two parts. In first one we take a stance that is more sympathetic to the Russellian view, arguing that Priest's arguments against it are inconclusive. In the second part, instead, we take a more sympathetic attitude towards Priest's objections. We argue, however, that if these objections pose insurmountable difficulties to the Russellian account (which is what one of the authors of this paper indeed thinks), then they pose the same difficulties also to Priest's favoured Hegelian account, and for the same reasons

1. Introduction

It is a fact that inconsistencies often arise in scientific theorizing; it is also a fact that these inconsistent scientific products are often useful. A reason for suspecting such products could not possibly be useful is the principle of explosion, which can be paraphrased as saying that from a contradiction everything follows. It is a theorem of classical logic that, if Φ is a wff of a language with an underlying classical logic, then from Φ and $\neg\Phi$, any arbitrary wff Ψ logically follows.

In the literature on inconsistent science, there are broadly three canonical approaches which attempt to square the usefulness of these products with their

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inconsistency. One approach (Vickers, 2013) seeks to argue, based on attention to historical case studies, that scientists often reason paying attention to only small sets of premises, carefully abstaining from employing doubtful premises as true assumptions rather than as useful means for inferring other propositions, while adhering to classical logic. Another type of approach (Brown and Priest, 2004) aims at providing formal models which are tolerant to inconsistency, either on account of the underlying logic or through other formal resources. A third type (Meheus, 2003) is focused on minute historical reconstruction of scientific episodes where *sui generis* maneuvers are employed to avoid triviality.

There is however a fourth approach, biting the bullet: the world is truly inconsistent i.e. an adequate account of the world makes room for true dialetheias, true contradictions which accurately represent some aspect of the world[3]. In this paper we shall focus on this type of approach. Specifically, we shall discuss Priest's account of change and motion, contrasting it with more orthodox rival, the Russellian account. The paper is divided in two parts. In first one we take a stance that is more sympathetic to the Russellian view, arguing that Priest's arguments against it are inconclusive. In the second part, instead, we take a more sympathetic attitude towards Priest's objections. We argue, however, that if these objections pose insurmountable difficulties to the Russellian account (which is what one of the authors of this paper indeed thinks), then they pose the same difficulty also to Priest's favoured Hegelian account, and for the same reasons.

Priest argues for worldly contradictions in order to give an account of two related difficulties:

i) The nature of the instant of change
ii) The nature of motion, a particular type of change: change of location with respect to time

We will first introduce the problem of the instant of change. Secondly we proceed to make the connection between the problem of the instant of change and the problem of motion explicit by linking them through the Leibniz Continuity Condition, henceforth LCC. Then we will argue that there are no good grounds for thinking the LCC holds in the cases Priest needs to bolster his

[3] Of course, Priest doesn't think just any old fact whatever obtains at the world. He rejects the metaphysical import of the principle of explosion in classical logic. For him dialetheias constitute a counterexample to this principle. (IC, p. 5)

position. We will then proceed to an examination of Priest's arguments against the orthodox account of motion, the Russellian theory of motion, showing how they can be answered. In criticizing Priest's arguments we will also elaborate on the metaphysics underlying the Russellian theory of motion, with the aim of making its explanatory resources explicit, this allows for a more satisfactory response to Priest's objections.

In the second part of this paper we discuss Priest's Hegelian account of motion in more detail. After showing why Priest thinks that his proposal meets the challenges raised against the Russellian account, we proceed to argue that the account suffers from the same difficulties of its rival.

2.The Hegelian Account of Motion: A Russellian Response

2.1 The Instant of Change and the Leibniz Continuity Conditio

Let us first introduce some jargon. Imagine, as in Priest's (1987) example of a pen being lifted from a paper, that an object is in contact with some surface before a time t and isn't in contact with that surface after t. Let s1 be the state "being in contact with the surface", which holds before t and s2 "not being in contact with the surface", which holds after t. The question of the instant of change for this problem can then be stated thusly: what is the state of the object precisely at t?

Priest thinks that there are four possible answers to this question (I.C., p. 160), which correspond to four possible types of change.

In changes of time A at the instant of change the object is found (only) in state s1. In changes of type B the object is only in state s2. Changes of type Γ involve the object being in neither state at the instant of change. In changes of type Δ, finally, the object instantiates inconsistently both states.

The concern here is that plausibly one of these types of change must take place, as this list appears exhaustive, yet neither option seems prima facie appealing. Let's review why:

i) Changes of type A and type B are asymmetric but there's no ready explanation for the asymmetry.
ii) Changes of type Γ, argues Priest, do not obtain here since conditions for having a specific state can be precisely stated and therefore do not involve vagueness so an analysis of negation delivers that one of s1 or s2 must always hold.

iii) Changes of type Δ involve worldly dialetheias, which is ultimately Priest's explanation of what's going on at the instant of change, however there's a reasonable presumption against using dialetheias in any explanation unless other options have been exhausted.

He supports the view that changes are of type Δ appealing on the one hand to problems with the other types of changes and on the other through appeal to the Leibniz Continuity Condition. Leibniz states this principle thusly:

> When the difference between two instances in a given series or that which is presupposed can be diminished until it becomes smaller than any given quantity whatever, the corresponding difference in what is sought or in their results must of necessity also be diminished or become less than any given quantity whatever. Or to put it more commonly, when two instances or data approach each other continuously, so that one at last passes over into the other, it is necessary for their consequences or results (or the unknown) to do so also. (Leibniz, 1687 as quoted in Priest, IC, p. 165)

The principle is meant to capture, in today's parlance, a property of sequences at their limits. Priest reads Leibniz as intending to give this principle a wide scope i.e. it is not only that sequences which fuse into each other share the same limits but also that whatever properties had by elements of the sequences are had also at the limit. The wide scope notion can be stated thus:

> LCC: Anything going on arbitrarily close to a certain time, is going on at that time too. (IC, p. 166)

Priest is quick to emphasize that the validity of the LCC need not be universal, for example, the limit of a sequence of rational numbers isn't a rational number. However, he believes that the LCC holds in some of the cases he puts forward as exemplifying type Δ changes, specifically he holds that for incompatible states s_α and s_β, and for different converging sequences fulfilling the LCC, where each s_α and s_β hold respectively throughout each of the converging sequences, they both hold at the limit.

In light of the LCC it's now possible to state precisely how motion, according to Priest, involves (Δ) type changes.

Suppose some particle satisfies the following equation of motion, $x(t) = ct$, where $x(t)$ is the position function, *t* a variable whose values are times and c some constant distinct from zero. Then, for some $t=t_\delta$, the position of the particle is given by $x(t) = ct_\delta$, yet before and after t_δ the position of the particle is such that $x(t) \neq ct_\delta$. By the LCC then, at t_δ it is the case that the position of the particle is both $x(t) = ct_\delta$ and $x(t) \neq ct_\delta$, a (Δ) type change which requires a worldly contradiction. What's more, assuming a smooth trajectory, each instant of the particle's trajectory is a limiting instant, from which it follows that at all times the particle is located and not located at some point![4]

Priest's support for the LCC turns on two assumptions:

LCC1: Structural features of our mathematical models don't match reality neatly, so that while at the instant of change, type A and type B changes remain mathematically possible given the atomistic features of our formal models, that doesn't guarantee that they're possible in reality.

LCC2: Nature exhibits robust connections i.e. the denial of Humeanism, so that there's a strong modal dependence of some temporal states on others.

Here's a possible reconstruction of his arguments that (Δ) type changes hold at the instant of change, which goes through both routes: via the LCC [premises i) and ii)] and via elimination by cases [premises iii), iv) and v)].

i) There are sequences of events, arranged in time, converging on an instant, such that either some proposition Φ holds throughout one of the sequences, while ¬Φ holds throughout the other or Φ holds throughout both sequences, while ¬Φ holds at the limiting instant (motion).
ii) Anything going on arbitrarily close to a certain time, is going on at that time too. LCC
iii) At the limiting instant in sequences such as those in premise i), (type A) either only Φ holds, (type B) only ¬Φ holds, (Δ) both hold or (Γ) neither.
iv) Assume either Φ or ¬Φ alone hold at the limiting instant. But both possibilities are equally arbitrary, therefore (Δ) both hold or (Γ) neither.
v) But (Γ) is not a possibility given Priest's account of negation and the

[4] this follows Priest's own example closely. (I.C., p. 170).

fact that conditions for one of these holding at the limiting instant can be precisely stated.

vi) Therefore, at the limiting instants of sequences such as those in premise i) (Δ) type changes occur. [By i) and ii)] and also by [iii), iv) and v)]

As evidence for the explanatory power of Δ type changes over the other types of changes Priest's lists broadly these troublesome cases:

Contact: Cases involving contact
Region: Cases involving motion between regions
Motion: The problem of motion generally

Premises i) and iii) are assumed truths about the scenarios under discussion. While i) has an air of plausibility to it, it is the case that many properties aren't had at instants, for example the property of being in the process of graduating or related properties. Still, so called intrinsic properties, such as being bent or certain relational properties such as being located at spatial region R are plausibly had at instants.

As for premise number iii) it is interesting to note that the disjoint states could hold without necessitating that either Φ or ¬Φ hold. Torza (MS) has proposed, in the context of the measurement problem in quantum mechanics, that the position operator can be interpreted in terms of a particle having some position even as it fails to necessitate that it has a specific position pre-measurement. Since Priest himself believes his account can make sense of quantum mechanical superposition, bringing up this possibility seems dialectically appropriate. Of course this view was introduced to deal with the quantum case, however, it appears just as suitable to deal with the problem of the instant of change as Priest's alternative. If some spread hypothesis is needed to account for motion, it might as well be stated in Torza's terms than in Priest's, since it doesn't require dialetheias.

As to LCC2 Priest himself admits Humeanism would reject it and Humeanism seems to be a live position in current philosophical debate (Sider, 2011; Weatherson, 2015). But even granting the falsity of Humeanism that still doesn't require Priest's dialethetias. Suppose some primitivistic account of the laws of nature is embraced consistent with primitive velocities (Maudlin, 2007; Tooley, 1988), such that there are necessary causal connections between properties. In the specific case of motion an example is afforded by primitive velocities. According to Tooley (1988, p. 237) primitive velocity is *"a theoretical property of an object at a time, and one that is causally related to an object's position at different times, rather than as a logical construction out of them"*. At the instant of change then objects could possibly instantiate these primitive velocities. This is a strange possibility, incompatible with the Russellian view, but for all that less strange than the possibility of worldly dialetheias.

Now as to iv), we think the strongest criticism one can make of Priest's move has to do with the fact that the type of properties he brings forward in his examples aren't the type of properties one might think of as fundamental. One could grant that for all space-time points there's a fact of the matter whether some fundamental property, such as a field value, is instantiated there. However, properties such as "being inside or outside", "being in contact with" and the like are too coarse grained semantically to distinguish between very close scenarios i.e. they determinately apply in some cases and there's no fact of the matter in borderline cases.

Imagine a world containing a circular figure surrounded by space. Let Φ be the property of being this circular figure and $\neg\Phi$ its complement. It's not clear what properties are picked out by Φ and $\neg\Phi$ in this context: do they pick out the circular figure together with its limiting points or the circular figure minus its limiting points? Both are equally good candidates, since Φ and similar properties, for all of Priest's remarks, aren't fine grained enough. Even if we suggest a new property Φ^* that picks out the circular figure together with its limiting points (or Φ^{**} without its limiting points), that would just introduce a convention.

Suppose there's a function which specifies the position of a point-object throughout an interval, initially the object is determinately inside the circular figure and it passes continuously to a region that is determinately outside. Then we ask: at the limiting region is the point inside or outside the circular figure? There's no fact of the matter, given that Φ is semantically coarse grained. We do no better if we ask with regards to Φ^* (or Φ^{**}). The answer would be yes (or no) in a purely conventional sense and hence arbitrariness is to be expected. There's a fact of the matter as to the location of the point at each time, but those needn't fix truths about non-fundamental properties. We can reason analogously in the LCC cases.

Consider Contact and Region. With regards to these examples it is also not altogether clear why we should accept Priest's diagnosis that vagueness of some sort isn't involved. Take properties such as "touching the surface" or "being inside or outside a room", while one may grant that there may be a fact of the matter as to the state of the world at those times, it's not clear there's a way of specifying that fact of the matter in the vocabulary Priest is employing here. For the case of a pen being lifted from a surface, given that pens are composed of material particles according to our best physics and that these interact through several different forces, it's certainly not clear that there can be contact between these objects in the sense intended by Priest[5].

But even if we abstract from the physical interactions and focus only on the properties of space-time, there's certainly nothing in our practices, or in the world which fixes the meanings of expressions such as "contact" in a unique way. There are broadly four ways allowed by spatio-temporal topology for the pen and the surface to be in contact:

1. The region occupied by the pen is open and that of the surface is closed
2. The region occupied by the pen is closed and that of the surface is open
3. Both are closed
4. Both are open

Notice that if we mind these formal structures all appearances of inconsistency disappear. As we emphasized above there's nothing in the expression "contact" which allows us to say that either of those possibilities must hold for contact to occur or even that contact requires only one instead of any of the possibilities. Moreover, case 3, which resembles case (Δ), doesn't require worldly inconsistencies: objects touch iff they intersect[6], but that is perfectly consistent. Similar things can be said for the case of some object crossing from one room into another as in Region, as illustrated above.

Finally, there's a notion of change in the neighborhood which is sufficiently explanatory and fits well with our current theories of persistence through time. The Russellian theory of change. [PoM (442)]:

> Change is the difference, in respect of truth or falsehood, between a proposition concerning an entity and a time T and a proposition concerning the same entity

[5]Van Inwagen (1990) makes a related point *vis a vis* contact as a sufficient condition for mereological composition.

[6]For a related discussion of the puzzles of continuity see Kilborn's *Contact and Continuity* in Zimmerman ed. (2007).

and another time T′, provided that the two propositions differ only by the fact that T occurs in the one where T′ occurs in the other.

Priest dismisses this cinematic account, yet the reasons he offers are those discussed above which, while possessing some strength can be better accounted for by the alternatives we laid down. According to this view, there is no state of change. This is a general case of the denial of states of motion to which we now turn to.

3. The Russellian Theory of Motion

The gist of the Russellian theory of motion[7] (Russell, 1903) is that motion is a relational property whose features, in classical space-time, are better understood in light of the structures of contemporary real analysis. Russell's main argument for his theory is that it accounts for all observable properties of motions[8], the formal apparatus required to state it is free of inconsistencies[9] and it can straightforwardly be viewed as a piece of applied mathematics suitably interpreted with an underlying metaphysics of time, as we will argue in what follows.

The standard Russellian account characterizes motion, rest and related notions in the following fashion.

i) An object X is in motion throughout an interval iff it occupies different positions at different times in that interval.
ii) An object X is at rest during an interval iff it occupies the same position at all times during that interval.[10]
iii) The time series as well as the spatial series i.e. the series of places, must be continuous.

[7]Also referred to as the at-at theory of motion. (Sider, 2001).
[8]In his *Our Knowledge of the External World* (1914) Russell argues that, even if points and instants turn out to be logical fictions, it is possible to construct suitable mathematical objects to play their role such that the formal theory of continuity retains its validity.
[9] "The chief reason for the elaborate and paradoxical theories of space and time and their continuity, which have been constructed by philosophers, has been the supposed contradictions in a continuum composed of elements. The thesis of the present chapter is, that Cantor's continuum is free from contradictions." (Russell, 1903, p. 352)
[10]While Russell's original presentation of his theory presupposes a Newtonian space-time with absolute positions, this thesis is wholly compatible with a Galilean space-time structure where only relative positions can be defined.

iv) Motion presupposes both change and the property of occupying a place at a time.[11]

v) Motion is a three-term relation between an entity, the time series and the spatial series. An entity can occupy the same place at different times, in those instances it is at rest, however it cannot occupy different places at the same time.[12]

vi) For motion to occur the entity must be at different places at neighboring times, however short. If it's in motion throughout an interval minus the end-point then it's ceasing motion, whereas if it's at rest throughout an interval, minus the end-point then it's beginning to move.[13]

In his own words. [PoM (446)]:

> Motion consists in the fact that, by the occupation of a place at a time, a correlation is established between places and times; when different times, throughout any period however short, are correlated with different places, there is motion; when different times, throughout some period however short, are all correlated with the same place, there is rest.

There's much to unpack here. It will however be better to unpack Russell's theory in light of Zeno's paradox of the arrow, an argument to which Russell gives high marks.[14]We will use here Paul Hager's (1987) excellent reconstruction of Zeno's paradox of the arrow, as formulated by Russell throughout his writings:

(1) Finite intervals consist of series of points and instants.
(2) The series of points and instants are either finite or infinite.
(3) The series of points and instants can't be infinite (since that leads to contradictions)

[11]Russell states in PoM (442) that motion consists in *"the occupation, by one entity, of a continuous series of places at a continuous series of times"* and that it presupposes the notion of *"occupying a place at a time"* as well as that of change.

[12]"Before applying these remarks to motion, we must examine the difficult idea of occupying a place at a time. Here again we seem to have an irreducible triangular relation. If there is to be motion, we must not analyse the relation into occupation of a place and occupation of a time. For a moving particle occupies many places, and the essence of motion lies in the fact that they are occupied at different times." Russell, PoM, (225).

[13]PoM (446).

[14]Russell regards Zeno's arguments as incapable of refutation without an adequate mathematical picture of the continuum and the infinite. He says as much in his *Mathematics and the Metaphysicans* (1917) and *Our Knowledge of the External World* (1914), as well as in his *Principles of Mathematics* (1903).

Therefore

(4) Finite intervals of spaces and times consist of finite series of points and instants. [1, 2, 3]
(5) But successive discontinuous occupation of points and instants is not sufficient to constitute the essential continuity of motion through such intervals.

Therefore

(6) The essential continuity of motion entails that a moving object must have throughout its motion (and hence at every instant and point) something to supply the continuity which an object at rest lacks i.e. a state of motion.
(7) At each instant the arrow in flight simply is where it is. (Zeno's platitude)
(8) An arrow that is where it is at an instant, does not move during the instant (otherwise the instant would have parts).
(9) But an arrow that does not move during an instant has no state of motion during the instant.

Therefore

(10) The arrow in flight has at each instant no state of motion. [(7), (8). (9)]
(11) The arrow has no motion. [(6), (10)]

(Hager, 1987)

Hager notes that Russell interprets Zeno's core argument as requiring (7), (8) and (9) which he takes to be true and whose entailment (10)[15] he also accepts. Where Russell disagrees is with regards to (9) and (11). Russell rejects (6) and the assumptions motivating it, that is (3), (4) and (5).

He rejects them on account of the modern mathematical theories of continuity and infinity, owed to Cantor, as well as the Cauchy-Weierstrass rigorization of the calculus in terms of limits instead of infinitesimals. The calculus as formulated by Newton and Leibniz was instrumentally adequate, however the use of infinitesimals and its imprecise notions of continuity and the

[15] "This static theory of the variable is due to the mathematicians, and its absence in Zeno's day led him to suppose that continuous change was impossible without a state of change." PoM (333)

infinite rendered its consistency suspect and hence its explanatory power was greatly diminished.[16] The modern account of the dynamical equations of motion is grounded on the coherent notion of the limit of a continuous function:

(i) $v(avg) = \frac{\Delta x}{\Delta t}$

(ii) $v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$

(iii) $a(avg) = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$

(iv) $v = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$

On his account of motion vis a vis Zeno's paradox of the arrow Russell says. [PoM (327)]:

> Weierstrass, by strictly banishing all infinitesimals, has at last shown that we live in an unchanging world, and that the arrow, at every moment of its flight, is truly at rest. The only point where Zeno probably erred was in inferring (if he did infer) that, because there is no change, therefore the world must be in the same state at one time as at another.

There is a sense to be made, mathematically, of notions such as instantaneous velocity and instantaneous acceleration, as the first and second derivatives (at any particular time) of continuous functions whose domains are sets of times and whose ranges are sets of spatial positions. Now, granting the consistency of the Cauchy-Weierstrass rigorization of the calculus as well as Cantor's characterization of the continuum and the infinite, this should suffice to show that the Russellian view is at least consistent with the possibility of motion, as characterized in this theory. Is the theory explanatory though? Does it explain what constitutes the motion of objects?

The answer to this question depends on one's metaphysical interpretation of the theory itself. What we mean by a metaphysical interpretation of the theory is an account of what the fundamental terms of the theory are meant to stand for in the world and what is their nature. For example: What is space? What is time? What is matter? In that respect, Russell's theory of motion already has an

[16] PoM, 303. See also from Peter Vickers *Understanding Inconsistent Science* (2013).

interpretation afforded by its author as we'll discuss below[17].

In the early chapters of his PoM Russell provides the following definition of applied mathematics. [PoM (9)]:

> Thus pure mathematics must contain no indefinables except logical constants, and consequently no premises, or indemonstrable propositions but such as are concerned exclusively with logical constants and with variables. It is precisely this that distinguishes pure from applied mathematics. In applied mathematics results which have been shown by pure mathematics to follow from some hypothesis as to the variable are actually asserted of some constant satisfying the hypothesis in question. Thus terms which were variables become constant and a new premise is always required, namely: this particular entity satisfies the hypothesis in question.

We don't need here to commit to particular details of Russell's philosophy of mathematics in order to lay down the metaphysical content of his theory of motion. The quote serves rather to illustrate the fact that Russell believes a theory, such as physics, to be interpreted in terms of particular objects, in the case of motion, enduring material particles in space and time. Also these objects satisfy the structural features of the formal theory.

Russell′s discussion of Newtonian Dynamics in PoM interprets the standard physical theory at the time as being committed to:

i) Eternalism:[18] The thesis that all times, past, present and future exist.
ii) Endurantism: [19] The thesis that objects persist by having different properties at different times.

[17]Russell makes much of the notion of interpretation *vis a vis* applied mathematics connecting it to problems in both physics and metaphysics. See for example Russell (1914), (1927) and (1948) for abundant examples.

[18]"The so-called predicates of a term are mostly derived from relations to other terms; change is due, ultimately, to the fact that many terms have relations to some parts of time which they do not have to others. But every term is eternal, timeless and immutable; the relations it may have to parts of time are equally immutable. It is merely the fact that different terms are related to different times that makes the difference between what exists at one time and what exists at another." PoM (443). See also *The Russellian Theory of Time* by Nathan Oaklander in his (2004).

[19]″The most fundamental characteristic of matter lies in the nature of its connection with space and time. Two pieces of matter cannot occupy the same place at the same moment, and the same piece cannot occupy two places at the same moment, though it may occupy two moments at the same place. That is, whatever, at a given moment, has extension, is not an indivisible piece of matter: division of space always implies division of any matter occupying the space, but division of time has no corresponding implication." PoM (440)

iii) Substantivalism about space:[20] The thesis that spatial points exist independently of matter.
iv) Substantivalism about time:[21] The thesis that instants of time exist independently of matter.
v) Dualistic substantivalism with regards to matter and space/time: The thesis that matter is something over and above space and time.
vi) The separability of space and time: The thesis that space and time are different types of objects: what Gilmore, Calosi and Costa call separatism (2016).

This is an explanatory metaphysics of motion, which interprets the pure formal theory in terms of temporal and spatial series coming together into a block universe, *an unchanging world*, where particular objects do change by having different properties at different times while they move according to dynamical laws occupying different positions at different times.

Now, the core of Russell's theory of motion can survive giving up several of those commitments without losing explanatory value. Russell himself would later give up substantivalism, separatism and endurantism. By the time of the publication of The Analysis of Matter he had embraced a unitist framework and had given up on endurantism, on substantivalism in all its forms as well as on separatism with regards to space and time and considered the properties usually attributed to matter as an example of attributing neat logical properties to an inferred entity; space and time are also inferred entities at this later stage, to be substituted by logical constructions out of classes of events. Eternalism on the other hand, seems necessary in order to characterize change in Russellian terms as well as in connection to his response to Zeno's arrow. It is the fact that the arrow has its positions in space and time *sub specie aeternitatis*, while being in different positions at different times, which allows a coherent response.[22]

Before moving on we want to comment on one explanatory virtue which emerges from combining the Russellian theory of motion, as he formulated it, with his later view on persistence, that is perdurantism as Russell himself did in

[20]See PoM. Chapter 58 for Russell's defense of substantivalism.
[21]"Among terms which appear to exist, there are, we may say, four great classes: (1) instants, (2) points, (3) terms which occupy instants but not points, (4) terms which occupy both points and instants. It seems to be the fact that there are no terms which occupy points but not instants." PoM (438)
[22]Boccardi (2017) mentions some of the difficulties inherent in the notion of passage which must be faced by those who embrace an A-theoretic account of time and change.

both The Analysis of Matter (1927, 1954; p. 246) and Human Knowledge (1948, p. 290).[23]

Perdurantism (4D) is the view that an object persists by having different temporal parts, each for any time at which the object exists. These instantiate properties directly and a fourdimensional object is said to have a property P iff it has a temporal part that is P[24]. In this theory the arrow is a fourdimensional space-time worm which has different parts at the different times at which it exists. The arrow moves at a time iff its temporal part at that time is such that its neighboring temporal parts at neighboring times are at different places. The arrow accelerates at a time iff the temporal part at that time is such that it belongs to a curved temporal segment of the arrow.[25]

In the eternalist, 4D account there is no miracle to be explained, contra Russell's own remark: *"It is never moving, but in some miraculous way the change of position has to occur between the instants, that is to say, not at any time whatever"*[26]. The world is just a static block universe populated with instantaneous temporal parts, distributed throughout space-time. The arrow itself is an unchanging fourdimensional object and talking about its motion is just an offhanded way of talking about the spatial distribution of its temporal parts at different times. Temporal distances are measures of the lengths of material paths in the static block universe. Not a mere mathematical technique, but a measure of an actual spatio-temporal length.

[23] "If the physical world is held to consist of a fourdimensional manifold of persistent moving particles, it becomes necessary to find a way of defining what is meant when we say that two events are part of the history of the same piece of matter. Until we have such a definition, "motion" has no definite meaning, since it consists of one thing being in a different place at different times. We must define a "particle" or material point, as a series of space-time points having to each other a causal relation which they do not have to other space-time points. There is no difficulty in principle about this procedure. Dynamical laws are habitually stated on the assumption that there are persistent particles, and are used to decide whether two events A and B belong to the biography of a particle or not. We merely retain the laws, then turn the statement that A and B belong to the same biography as a definition of "biography", whereas before it seemed to be a substantial assertion." (Russell, 1948, p. 290)

[24] In recent articles Macías Bustos (2016); Calosi and Fano (2014) have commented on the explanatory gains from making this move, but while the former believes this is a natural extension of the Russellian view, the latter authors think this view, 4D-Motion, should be a new contender. As we pointed out Russell (1927, 1948) did in fact extend his theory of motion this way.

[25] From the standpoint of that temporal part's rest frame. This way of putting it is more in line with the structure of Neo-Newtonian space-time and therefore with Russell's unitism, which he embraces as far back as 1914. Neo-Newtonian space-time has no absolute velocities, since it lacks the structure necessary to speak of different distances at different times. There's only rest and velocity from a given reference frame, though there are absolute accelerations. See Maudlin (2012).

[26] Russell, 1929.

4. Priest's Objections Against the Russellian View

Priest's objections against the Russellian view can now be stated precisely and answered in light of the previous discussion.

i) The Russellian view is incompatible with Laplaceanism

A Laplacean world is one in which its state at one time determines its states at all times. Classical physics textbooks (Landau and Lifshitz, 1982) tell us that, for classical closed systems, the positions of all particles together with their velocities determine a unique trajectory in phase space, the shortest one, between that time and another: unique evolution. However, Russellian velocities aren't instantaneous properties, they're neighborhood properties, requiring more than one time to be specified.

Now, while it's strictly true that Russellian worlds aren't Laplacean worlds, in the metaphysical sense i.e. their states at any one time are grounded in their states at neighboring times, that is not a problem for the Laplacean view methodologically speaking. Having discovered the state of a classical system at a time as a function of its state at neighboring times it's then possible to take the positions and velocities at that time and find the shortest route through phase space between that time and another, giving us unique evolution. Lapleaceanism would be a priori false, but its methodological import as a normative ideal in physics remains in place.[27]

ii) Zeno's arrow reloaded

Priest argues that the Russellian theory is unsatisfactory as a response to Zeno. He restates the argument followed by the claim that while the mathematics works, it's just implausible to think it's explanatory. How can a sequence of positions at instants add to a temporal journey? Each position has zero temporal length yet they constitute a non-zero temporal journey. However, there's an

[27]Hughes (1989, p. 76) has argued that for classical mechanics to be Laplacean, in the metaphysical sense, unusually strong constraints are needed: unique temporal solutions and continuous differentiability of the system's Hamiltonian for all position and momentum values, for all possible states of the system. This would rule out Democritean worlds were incompressible spheres exchange forces through contact through loss of differentiability. Possibilities: we gain some, we lose some.

analogous puzzle with regards to space: if objects are constituted by point-like parts, of zero-spatial length, how can a set of them constitute an object's non-zero volume? In the spatial case it's easier to accept that the geometry of space has these mathematical features, since all spatial positions are given[28]. However, as we discussed previously, Russell's theory presupposes eternalism, all times are given. The world is there sub specie aeternitatis and the worldlines of particles are already given: the particles positions through time are given. The mathematics allows for the calculation of the length of some particle's worldline from one time to another or in the 4D case, the length of some proper space-time subregion of the particle.

Eternalist space-time has a given structure, including a metric, which accounts for the fact that there are lengths between any two times at which the arrow exists without the need to posit anything further that takes the arrow from one point of space to another. Of course, making sense of all this requires strongly embracing the analogies between space and time (Sider, 2001, p. 87). That is no small thing for those who strongly feel time is different from space. However, inasmuch as one is comfortable with these analogies the notion of a distance in space-time makes as much sense as the notion of a distance in space. And as Gilmore, Calosi and Costa (2016) emphasize, our best theories of space and time are more parsimoniously stated in a unitist framework which treats both as different aspects of a single object: space-time.

5. The Hegelian Account of Motion: a Sympathetic Criticism

In the previous sections, we have discussed Priest's arguments for inconsistent motion together with a sympathetic exposition of the Russellian conception of motion. After briefly rehearsing Priest's views, we now introduce Priest's own "Hegelian" proposal from a standpoint more critical of the orthodox account. The remains of this paper is dedicated to a critical assessment of this proposal.

5.1 The spread hypothesis.

Consider an object O whose trajectory through space can be represented by a continuous and differentiable cinematic function, $f(t)$. The domain D of the cinematic function (isomorphic to a subset of the reals) represents a set of

[28]Relationalists would reject this though. (Nerlich, 1994)

instants of time, while the values of f at each of these instants represent the locations in space of the object at that time. Following Priest, let us write Bx for 'b is located at point x'. Let us also suppose that each real, r, has a name, $\underline{r}$. Then the evaluation, v, which corresponds to this motion according to the Russellian account, is just that given by the conditions (IC, p. 177):

(1a) $1 \in v_t(B\underline{r})$ iff $r = f(t)$

(1b) $0 \in v_t(B\underline{r})$ iff $r \neq f(t)$

Hegel, who is the main source of inspiration for Priest's theory of motion, notoriously held the view that motion involves the instantiation of contradictory states of affairs (IC, p. 175):

> [M]otion itself is contradiction's immediate existence. Something moves not because at one moment of time it is here and at another there, but because at one and the same moment it is here and not here . . .

According to Priest, the rationale of Hegel's account rests on the observation that objects in motion can't be localized with absolute precision.

> So let us inquire why, exactly, Hegel held this view of motion. The reason is roughly as follows. Consider a body in motion—say, a point particle. At a certain instant of time, t, it occupies a certain point of space, x, and, since it is there, it is not anywhere else. But now consider a time very, very close to t, t'. Let us suppose that over such small intervals of time as that between t and t' it is impossible to localize a body. Thus, the body is equally at the place it occupies at t', x' ($\neq x$). Hence, at this instant the body is both at x and at x' and, equally, not at either. This is essentially why Hegel thought that motion realizes a contradiction [IC, p. 176].

Before discussing the reasons we have to believe that objects should not be localizable at instants of time, let me proceed to introduce Priest's concrete proposal. Drawing from Hegel's suggestions, Priest supposes that at each instant t of their journey, objects in motion are not only localized at the position corresponding to the value of their cinematic function at that time, $f(t)$, but also at an entire (small) interval of positions about that location. Thus, for each time t, there is an interval containing t, θ_t such that, if $t' \in \theta_t$, b's occupation of its location at t' is "reproduced" at *t*. Priest calls this idea: *the spread hypothesis*. In our semi-formal language, the Hegelian evaluation is given by the following formulas:

(2a) $1 \in v_t(Br)$ iff, for some $t' \in \theta_t$, $r = f(t')$

(2b) $0 \in v_t(Br)$ iff for some $t' \in \theta_t$, $r \neq f(t')$

One can immediately check that the satisfaction of these conditions sometimes involves the instantiation of contradictions. This is why. Let Σ_t stand for the spread of all the points occupied by object O at t: $\Sigma_t = \{f(t')|t' \in \theta_t\}$. If Σ_t is non-degenerate, then for all $r \in \Sigma_t, 1 \in v_t(Br \wedge \neg Br)$.

On the contrary, if Σ_t is degenerate no contradiction is instantiated at t. As noted by Priest (I.C., p. 178) this may happen for either of two reasons: either (1) if θ_t is itself degenerate, i.e. if $\theta_t = \{t\}$ (in which case the Hegelian description relapses into the Russellian one); or (2) if the cinematic function is constant within θ_t. Either way, objects are in motion at a time t if and only if Σ_t is non-degenerate.

Before discussing how this account of motion is supposed to overcome the difficulties that Priest has raised against the Russellian conception, let me conclude this brief outline with some remarks about the presumed size of the spread and about its direction.

If Priest is right, if an object is moving at time *t*, then its position (at *t*) is spread over a space interval (Σ_t). We have seen that when the velocity of the object is 0 at *t*, then the size of Σ_t at *t* is also 0. How large is this interval when its velocity is not 0? Priest prefers to remain as neutral as possible as to how to describe the exact relation between the size of the spread, $\sigma(\Sigma_t)$, and the instantaneous velocity. However, he makes a couple of comments about it, which give us an insight about the thrust of his Hegelian account, and that we shall further discuss in the following pages.

First, he claims that "[i]t is quite plausible to suppose that its length depends on the velocity of b, so that the faster b is going the more difficult it is to 'pin it down'" (I.C., p. 178). While it is reasonable, given the explanatory purpose of the account, to assume that there is a positive correlation between the size of the spread and the speed of the object, the expressions "difficult" and "pin it down" in the comment just quoted invites us to suspect that epistemological considerations play an important role in Priest's understanding of his own account. This impression is further confirmed by the other comment Priest makes about the size of the spread.

> Perhaps the measure of Σ_t, $\sigma(\Sigma_t)$, just is the uncertainty in the location of the object at *t*. Perhaps quantum mechanical indeterminacies are fundamentally the result of inconsistencies in motion, and in particular in the spread postulated by the spread hypothesis. This suggestion at least allows us to give physical significance to the spread. (Priest, p. 180)

This is not the place to discuss the notoriously complicated role of epistemological considerations in the foundations of quantum mechanics. It is hard, however, not to suspect that the notion of an observer plays an indispensable role in this picture. Moreover, it seems to us that linking the size of the spread to quantum indeterminacy is incompatible with the first assumption, that the size of the spread would increase with the speed of the object. As Priest himself notices (I.C., p. 181), in fact, in general, the momentum, *p*, of an object is in a continuous state of change too; hence, by exactly the same considerations, it is spread out at t over a range Π_t. Heisenberg's uncertainty principle then gives us that:

$$\sigma(\Sigma_t) \times \sigma(\Pi_t) \geq \frac{h}{2\pi}$$

We believe that this interpretation of the spread hypothesis is untenable. First, contrary to what is suggested by Priest, the amplitude of the quantum indetermination of momentum does not depend on how "fast" momentum is changing (i.e. on the magnitude of the force applied to the particle).

Worse still, the standard deviations of the probabilities associated with position and momentum, as indicated by Heisenberg's uncertainty principle, have an inverse relationship, or one which is bounded from below. This entails that as the precision with which the momentum of the particle can be determined increases (which in the Priest's proposed interpretation would correspond to its velocity approaching a constant value) the precision with which its position can be determined decreases. This cannot be. To see why, consider the (limit) case in which the velocity (hence the momentum) of the particle is determined with infinite precision to be equal to 0. Heisenberg's principle dictates that, in this circumstance there is maximum indeterminacy of the particle's position, which, under Priest's suggested interpretation, would correspond to a maximum size of the spread $\sigma(\Sigma_t)$, hence with a minimum of accuracy. This conflicts with the whole rationale behind the Hegelian account, according to which an object at rest can be localized with infinite (or maximal) accuracy.

Since nothing of importance hinges upon the quantum interpretation of the spread hypothesis, we shall not press this point further here.

The other detail of the proposal that we shall touch on regards the direction of the spread θ_t. Should we think that it extends in both directions about t, or not? Priest argues that we should think of it as extending only in the direction of times earlier than t. This is supposed to avoid suspicious cases of backward causation.

The thrust of the argument is this. If the spread θ_t extended to times later than *t*, then the instantaneous state at *t* would depend (in part) on the trajectory of the object at times later than *t*. But such trajectory could well depend on events that happen after t. Thus, the state at *t* would come to depend (in part) on what happens after *t*. Priest himself does not think this argument is conclusive (the dependency in question might be non-causal) and we shall not take issue with it here.

5.2 The argument from indistinguishability

Priest claims that the Hegelian account has significant advantages over the Russellian one. In particular, he thinks that it scores better at responding to two challenges that we have already discussed. We may call these, respectively, the objection from indistinguishability and the objection from incremental accretion. We have already discussed how the Russellian may respond to them. Here we shall briefly rehearse them in turn. In the rest of this paper we shall argue that, even conceding that these objections pose serious difficulties to the Russellian stance (which one of the authors of this work indeed thinks they do), the Hegelian account does nothing to alleviate them.

Priest exposes the objection from indistinguishability as follows:

> It follows from the [Russellian] definition that there is no such thing as an intrinsic state of motion. If one had a body in motion and took, as it were, a logical "picture" of it at an instant, the picture obtained would be no different from one of the same body at the same place, but at rest. Of course, an object in motion can have an instantaneous non-zero velocity, but it would be wrong to think that this differentiates it intrinsically from a static body. (I.C., p. 196)

Russell, we have seen, was ready to admit that his picture of motion entails that there is no *intrinsic* state of motion. In this sense, remember, he conceded that at each instant the arrow is truly at rest. What is the theoretical cost, if at all, of making this concession? Why should we think that the distinction between rest and motion be grounded on a distinction at the level of the world's *temporary intrinsic* features? Why should the *instantaneous* state of a changing universe be any different from the *instantaneous* state of an unchanging universe which happens to be (always) in that very same state?

This admittedly strange idea, according to which an object can move without ever being in an intrinsic state of mov*ing* had its enemies since when it was conceived. This is, for example, how James expresses this worry:

> whatever motion really may be, it surely is not static; but the definition we have gained is of the absolutely static. It gives a set of one-to-one relations between space point and time-points, which relations themselves are as fixed as the points are. It gives *positions* assignable ad infinitum, but how the body gets from one position to the other it omits to mention. The body gets there by moving, of course; but the conceived positions, however numerously multiplied, contain no element of movement, so Zeno, using nothing but them in his discussion, has no alternative but to say that our intellect repudiates motion as a non-reality.[29]

These complaints appear to stem from the intuition that the banishment of instantaneous states of change would make the world capricious and change unaccountable for. Doubtless, for example, instantaneous velocities feature in many standard scientific explanations. The challenge we are discussing is to explain how instantaneous velocity can fulfil the causal role that is ascribed to it by standard practices in the physical sciences, if it is not part of the intrinsic, instantaneous state of the universe at a time. Standard physics textbooks tell us that the state of the universe at any time is determined by the state at any previous time plus the laws of physics. The state of the universe at a time *t*, we are told, is given by the positions of all the particles at *t, plus their velocities at t*. But, according to the Russellian view, the instantaneous velocities of the particles supervene solely on their positions at various times preceding and succeeding *t*, hence they add nothing to the state of the universe at exactly the time *t*. Now, the positions of particles at *t* have clearly no effect whatsoever on the positions in the immediate future of *t*. How could their velocities have any greater explanatory

[29]James (1987, p. 735)

role, if they supervene solely on these positions themselves?

Priest argues, along similar lines, that the Russellian account rules out determinism *a-priori*:

> Suppose that the universe were a Laplacean one, in which the state at any time is determined by the state at any (prior) time. Then the orthodox account of change would be impossible. For the instantaneous state of an object (or of all objects) cannot even determine whether it is at the same or at a different place at subsequent times. (Recall that the velocity–or momentum–of an object is not determined by its intrinsic instantaneous state.) Now I am certainly not insisting that the universe is Laplacean. It is not. But it is a curious theory that rules this out a priori (I.C., p. 174).

In response to the argument from indistinguishability, some authors [30] developed theories of instantaneous velocity which depart from Russell's deflationary understanding in various ways. Tooley, for example, defended a view that "treats velocity as a theoretical property of an object at a time, and one that is causally related to an object's position at different times, rather than as a logical construction out of them". [31] Lange suggested that "classical instantaneous velocity is something like a dispositional property, a tendency, a power, or a propensity. To be moving at t_0 in a given direction at 5 centimeters per second is to have a certain *potential* trajectory."[32]

Priest too is among the proponents of an intrinsicalist account of motion. To be in a state of motion, according to him, we have seen, is to be inconsistently localized at a spread of different positions. How is the spread hypothesis supposed to alleviate this worry? Here is how Priest thinks it does (I.C., p. 180):

> This is obviously no problem for the Hegelean account. For it, there is an intrinsic state of motion: a certain inconsistent state. The difference between a body genuinely in motion and one changing place but at rest each instant is exactly that between a Hegelean state description and the corresponding Russellian one.

Let us grant that the Hegelian account provides us with an intrinsic

[30]e.g. Lange 2005; Arntzenius 2000; Tooley 1988; Bigelow and Pargetter 1990; Carroll 2002
[31]Tooley 1988: 237.
[32]Lange 2005: 453.

distinction. The account, remember, has it that bodies in motion at an instant are multiply located *at that instant*, while bodies at rest are not. This surely counts as a distinction in the intrinsic state of the object. But the predicament under discussion was not to provide an account which makes some (any) intrinsic distinction between motion and rest. Not *any* distinction would foot the relevant explanatory bill. To see what explanatory job instantaneous states of motion are called to do, consider the following argument by Arntzenius:

> Consider a ball moving from right to left through some region in space, and a qualitatively identical ball, perhaps the same one, some time later, moving from left to right through that very same region. If the full state at a time of a ball does not include an instantaneous velocity then the full state of the two balls is exactly the same when they occupy the same region. [...] *Why* does the one ball subsequently move to the left and the other ball subsequently move to the right? Surely it is conservation of velocity, or something like that, which determines that the one ball will keep moving to the left and the other will keep moving to the right. But if there is no such thing as instantaneous velocity, as there is not in the at-at theory, then *why* do the balls continue their motions in different directions?[33]

Now, if a ball is moving at time *t*, according to the Hegelian account, on top of being located at *f(t)*, the ball is *also* located at all points in Σ_t. The ball at rest, instead, is *only* located at *f(t)*. Why should the ball realizing the Hegelian description keep moving? Granted that the mere fact of being located at *f(t)* is explanatorily powerless with respect to the location of the ball after *t* (or before *t*, for that matter). Why should the addition of other locations for the ball at that same instant provide us with a dynamic element which eludes the cinematographic picture of motion?

Sure, if the locations contained in the spread Σ_t include locations of the ball at times *after t*, then there would be a clear connection between the instantaneous state of motion of the ball at *t* and its future trajectory. The instantaneous state would contain information about the future whereabouts of the ball. But such information does not help us to the least in understanding *why* the ball should leave the (spread out) location it occupies at *t*.

To simplify matters, consider a ball whose cinematic function in the identity function: $f(t) = t$. Suppose the size of the spread θ_t is one unit of time throughout its journey: $\sigma(\theta_t) = 1s$. At each time during its journey, the ball is

[33] Ibid.: § 2, our emphasis.

therefore inconsistently located at a spread Σ_t whose size is, say, 1m. At t=0 the ball is located at a spread Σ_0. As time goes by, the ball will occupy (inconsistently) a sequence of spreads: $\Sigma_{t>0}$. Why does the ball changes the locations that it (inconsistently) occupies? It seems like, in answering this question, the Hegelian finds herself in the same predicament as the Russellian.

Of course one could point out that the cinematic function $f(t)$ incorporates information about the whereabouts of the ball at *all* times, and it is in compliance to this function that the ball acquires the locations that it does at subsequent times.[34] But this is a response that is available (if at all) to the Russellian too: if the mere "compliance" with the cinematic function could answer the worry about the capricious behavior of the career of objects through space, then we wouldn't find ourselves in the predicament we are discussing in the first place, and the Hegelian account would thereby lose its chief allure.

One can bring this point home as follows. The locations that the ball may inconsistently occupy at any time are totally unrelated to the locations it may occupy at any other time. The nexuses, in both the Hegelian and the Russellian accounts, are provided solely by the cinematic function (which features essentially in the Hegelian description just as it does in the Russellian one). Why should the instantiation of inconsistent states of affairs as to the location of the ball at a time "force" it, or "induce it" or "dispose it" to be elsewhere at other times? If the ball occupies a spread of locations at a time, then it is *already* also "elsewhere" than where the cinematic function says it is at *that* time. But what does this tell us about the (consistent or inconsistent) whereabouts of the ball at any *other* time?

Summing up, if the Russellian account suffers from the above mentioned explanatory deficiencies, then the Hegelian account does too, and by the same token.

5.3 The argument from incremental accretion

Let us now turn to the other objection which Priest raises against the Russellian account and which he thinks the Hegelian account has the resources to tackle. As we have already discussed in the first part of this paper, the predicament is

[34]Incidentally, moreover, note that if (as suggested by Priest) the spread associated with the object at times t is restricted to times preceding t, then the Hegelian account would not even provide the appearance of a solution, for the intrinsic state of the object at t would contain no information whatsoever as to its whereabouts at times after t.

that of explaining how an object's journey through space could be composed of a sequence of going nowheres. This is how Priest expresses this worry (I.C., p. 174):

> Consider a point-object in uniform motion from x to y, say the tip of an arrow. And consider an instant of its motion, t_0. At t_0 the arrow advances not on its journey to y. (If it did make some headway, this would take time. The temporal stretch involved would not, therefore, be an instant.) Thus, at $t = t0$, total progress made equals zero. But a temporal interval, [x, y], is made up of such points. It would therefore seem that, since no progress is made in any basic part of the interval [x, y], no progress can be made in the whole. That is, the arrow never makes any progress on its journey at all. This is absurd.

Indeed, preoccupations such as the ones expressed by Priest here are often looked upon with certain contempt:

> these days no one worth his salt thinks instants "add up to" periods this way. If there are instants, periods are instants with distance-relations between them. The relations, not their relata, account for periods' extension: that is why (in the paradox) without putting distance-relations between the points, we don't get an extension.[35]

However, that one can define mathematical structures which defy this intuition does nothing to ease the discomfort. Mathematical structures are changeless (timeless) abstract entities, whose existence is often enforced by decree. An adequate description of physical reality is an altogether different matter. Even granting that the orthodox mathematical response is correct, it only tells us *that* the argument fails, but it doesn't tell us *where* it fails. As Priest rightly put it (I.C., p. 175):

> That one can prove a small mathematical theorem or two is one thing; but it does not ease the discomfort that one finds (or at least, that I find) when one tries to understand what is going on physically, when one tries to understand how the arrow actually achieves its motion. At any point in its motion it advances not at all. Yet in some apparently magical way, in a collection of these it advances. Now a sum of nothings, even infinitely many nothings, is nothing. So how does it do it?

[35] Leftow (2014, p.239).

According to Priest the Hegelian account eludes this difficulty. This is how it is supposed to work (I.C. p. 180):

> The Hegelian account of motion may be taken to locate a fault in the argument, but at a point different from that upon which Russell lights. For, according to Zeno's argument, at a particular point in time the object occupies only a single point in space, whence it follows that it advances not on its journey during that instant, i.e. that the measure of the set of points occupied at that instant is zero. Given the spread hypothesis, however, it is not true that the moving body occupies only a single point. At an instant, t, it occupies all the points in Σ_t, which is, in general, not a singleton. Indeed, provided the function of motion, f, is continuous, Σ_t is an interval, and therefore has non-zero measure. Thus, advance is made during a single instant, and hence during the aggregate of instants.

We think Priest is right in his contention that mathematical solutions, *per se*, do nothing to alleviate the difficulties posed by Zeno's arguments. However, we think that the Hegelian solution is deeply problematic, for reasons that we have already discussed in the first part of this paper. We argued above that the multiple location of a particle throughout a spread Σ_t of positions is silent as to the displacement of the particle through time. What connects a Hegelian state description to motion, we have argued, is the cinematic function alone, just like it happens according to a Russellian account. Analogously, Priest is entitled to call a given spread of locations Σ_t an "advancement" only if the description could suffice to justify why the object transverses a continuous sequence of such (inconsistent) locations as time goes by. We have argued that it does not.

One can best appreciate the source of this difficulty by advancing a Hegelian friendly version of the arrow paradox. According to the Hegelian account, objects move by occupying (inconsistently) different spreads of locations at different times. At each moment during its journey, an object occupies a region of space the same size as its spread Σ_t. During each instant *t*, the spread Σ_t of locations occupied by the object advances not at all. Then how does it manage to advance over a finite interval of time, given that this is constituted by nothing but a sequence of such spreads?

Another way to bring this point home is by following Priest in his suggested quantum interpretation of the spread hypothesis and then advance a quantum version of Zeno's paradox. Consider the quantum state description of a moving particle provided by the wave function in position space:

$$\Psi(x,t) = \frac{1}{2\pi h} \int_{-\infty}^{+\infty} A(k) e^{ikx-\omega t} dk$$

Whether we interpret this state as entailing that at each instant *t* the particle is inconsistently occupying multiple locations or not, Zeno's argument appears to go through unscathed. Over the finite interval of time $[t_0, t_1]$, the state of the particle changes from $\Psi(x, t_0)$ to $\Psi(x, t_1)$. But at each time during its journey the state of the particle changes not at all. How is it, then, that in a sum of *changing-not-at-alls* the particle manages to instantiate a different state?

5.4 Non standard calculus to the rescue?

There have been suggestions in the literature that non-standard variants of infinitesimal calculus might be put to use in responding to Zeno's paradoxes. There are interesting similarities between these proposals and Priest's Hegelian proposal. A brief discussion of them might help clarify our general worry with the Hegelian account.

Russell seems to have taken the rigorization of calculus by Cauchy and Weierstrass not only as allowing for, but also as commanding his at-at theory of motion. The crucial element behind this commandment appears to have been Weierstrass' "banishment" of infinitesimals.

> It is to be observed that, *in consequence* of the denial of the infinitesimal, and in consequence of the allied purely technical view of the derivative of a function, we must entirely reject the notion of a state of motion. The rejection of velocity and acceleration as physical facts (i.e. as properties belonging at each instant to a moving point, and not merely real numbers expressing limits of certain ratios) involves, as we shall see, some difficulties in the statement of the laws of motion; but the reform introduced by Weierstrass in the infinitesimal calculus has rendered this rejection imperative.[36]

Russell thought that, had it not been for its denial of infinitesimals, the adoption of Weierstrass rigorization of calculus might not have rendered the rejection of velocity as a physical fact "imperative". We now know that *it is* possible to rigorize calculus without banishing infinitesimals. It is therefore

[36] Russell, 1903, p. 473, our emphasis.

natural that we should ask whether the adoption of these non-standard models of analysis might allow us to derive less dramatic responses to Zeno's arrow argument. Indeed, the adoption of non-standard calculus in addressing Zeno's paradoxes has been advocated from a number of quarters (e.g. McLaughlin & Miller 1992). In spite of its intuitive appeal, I shall argue, the application of non-standard analysis to Zeno's paradox of the arrow provides no relief to the spread-theorist.

One may put the general worry about the use of calculus in solving Zeno's paradox in the following terms. Relations can be causally efficient only if some of the *relata* are independently causally efficacious. Since locations in space at a time are not individually causally efficacious, these arguments go, then neither could instantaneous velocities, since under its standard conceptualization velocity is a relational (or neighborhood) property, whose *relata* are precisely the positions of the object at different times.

Non-standard analysis is based on an enlargement of the set of real numbers R. The details of such expansion need not concern us here. Suffices to say that the result of the expansion is a set R some of whose elements correspond to the reals (the standard numbers). The other elements of R are called non-standard.[37] Among non standard numbers there are *infinitesimals* and *infinite* numbers.[38] Finite numbers are all the standard real numbers together with all the non-standard ones that are not infinite. A *monad* of a finite number *n* is the set of all finite numbers that are infinitesimally close to it. Each monad contains exactly one standard number (the difference between any two standard numbers is always a standard number). With this structure in place, one can define the familiar notions of calculus in a new and rather intuitive way. What concerns us here is the notion of derivative.

In non-standard analysis, $v(t_0) = \frac{dx}{dt}(t_0)$ iff $\frac{x(t)-x(t_0)}{t-t_0} - v(t_0)$ is infinitesimal for all t in the monad of t_0.[39] Does this definition make the value of the derivative an *intrinsic* property of the object? Arguably not. The definition above suggests that, if we adopt a non-standard model, we should say that objects occupy a single (standard or non-standard) position for each (standard or non-standard) time: if $x(t)$ was defined only over standard times, then the definition

[37] Any statement true in R is also true in R*, so R* is a model of analysis.
[38] Infinite numbers are numbers greater than all real standard numbers. Infinitesimals are inverse of infinite numbers.
[39] Robinson 1966: 68.

above would not make sense. Under this interpretation of what it means to occupy a position at an instant, it is clear that non-standard calculus offers no escape to Zeno's argument. The argument applies unscathed to the new hyperreal locational framework.

The other option is to interpret "points of space" and "instants of time" as entire hyperreal monads (infinitesimal spreads, as it were). If derivatives represent relational properties in the hyperreal locational framework, they arguably represent "instantaneous" monadic properties, if instants are conceived of as whole monads. Instantaneous velocity, in fact, can be thought of as the size of a monad of displacement divided by the size of a monad of time. Contrary to Zeno, it can now be said that the object does not occupy an amount of space equal to itself at each instant (monad) of time: over an instant (monad), the object occupies a space that is a monad of space greater than itself. Of course, the size of the object at an instant, as measured by a standard real number, will be the same. But it appears that we have escaped Zeno's (and Russell's) conclusion that, at any instant of time, the object moves not at all. During a monad of time, the object moves by a monad of space!

However, this is only the appearance of a solution, for the object advances by a monad of space *only if it occupies each hyperreal location in due turn*. Thus, unless it be claimed that at a single durationless hyperreal instant of time objects can advance by a monad of space, non-standard analysis will be utterly powerless in addressing the difficulties discussed in this paper.

Conclusions

Part of the reasons that Priest offers in favour of his Hegelian account of motion stem from a criticism of the Russellian account, together with a number of arguments purported to show that the Hegelian account allows us to overcome these difficulties. This paper is divided in two parts. In the first one we explored a number of ways in which the Russellian can resist these charges. In the second part, instead, we sympathize with Priest's criticism of the Russellian view but we take issue with the presumed superiority of the Hegelian account.

In this brief note we could not do justice to the impressive number of subtle and interesting issues that Priest's work raises. We have argued, however, that the Hegelian account as it currently stands remains unconvincing.

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