Commentary

General Relativity from A to B[*]

Robert Geroch

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James Owen Weatherall [**]

weatherj@uci.edu

Bob Geroch's *General Relativity from A to B* is, as its title suggests, an elementary book – the first word, rather than the last, on General Relativity (GR). This, I take it, is what makes the richness of the book so remarkable. As Geroch says in his preface, this book is not "a view from below [...] of a tower shrouded in mystery". Instead, it shows GR as it «actually works» (p. VII). Geroch reveals the nuts and bolts of the theory without getting bogged down in, or even introducing, the often complicated formalism of differential geometry. The book is a proof of concept: Geroch ably demonstrates that a detailed, precise, and yet fully accessible introduction to an advanced topic in physics is possible after all. One often hears authors and physicists, especially in the popular press, note the *beauty* or *elegance* of GR, and there is much in the structure of the theory to support such judgments. But what Geroch reminds us is that GR is also a *simple* theory. An advanced high school student could walk away from this book fully equipped to make predictions about the most exotic space-times one might think of.

General Relativity from A to B is not the kind of book that one responds to, in the sense of argue with. There is little to agree with or disagree with, here. It is also a recent enough book that little is to be gleaned from studying its historical context. Instead, my focus in this commentary will be on the feature of Geroch's discussion that permits him to say so very much, so precisely, but with essentially no technical formalism: the space-time diagram. There are three modest remarks that I want to make about Geroch's use of space-time

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[**] Department of Logic and Philosophy of Science – University of California

diagrams, which I will approach in turn. I do not take any of these remarks to be shocking or groundbreaking. I simply want to draw attention to how powerful these diagrams can be, both as computational tools and as a way of probing at least some foundational issues in gravitational theories.

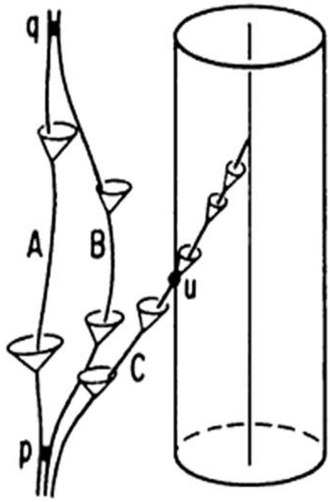
The first remark is methodological, concerning how one should approach problems in GR. In solving such problems, Geroch writes:

> The pattern in every case is the same. One first elicits a detailed statement of the actual physical experiment to be performed, complete with the measurements to be taken. One then represents the experiment by a space-time diagram. (pp. 155-156)

One then goes on to solve the problem, using the space-time diagram thus constructed. It is easy to take this suggestion in some limited way, perhaps as follows: when first approaching elementary problems, one should construct a space-time diagram. More difficult problems (naturally) should be approached using more sophisticated methods. But to take the suggestion in this way seems to me to be a mistake. I think Geroch has something more general in mind.

Geroch uses space-time diagrams in the fashion common to elementary treatments of relativity theory: he uses them to describe simple experiments in which different observers measure lengths, or durations, or send signals to each other. But he *also* uses them, even more effectively, in addressing the most advanced topic in the book: the physics of black holes. Here he proceeds exactly as he recommends in the quoted passage. He begins by giving a general account of the space-time under consideration, by describing (diagrammatically) the light cone structure of the Schwarzschild solution. He then describes a number of experiments that one might perform: he imagines an observer traveling towards the event horizon of the black hole and then turning back; an observer passing through the event horizon; an observer sending and receiving signals as she approaches the event horizon; etc.

An Example: Black Holes



Three observers in our Black-Hole Space-Time. Observer A remains in the distant external region, B ventures somewhat closer to the hole, and C passes through the horizon to the interior of the hole.

Illustration 1: An example of a space-time diagram from the black hole chapter of General Relativity from A to B. *The boundary of the cylinder represents the event-horizon of the singularity, represented by the line at the center of the cylinder. At point u, the observer with wordline C passes the event horizon. One can see the light cones along C lean increasingly towards the singularity so that at u, there is no future directed timelike curve that leaves the cylinder. Geroch uses such diagrams to explain the physics of complicated space-times without introducing detailed formalism.*

In each case, rather than proceed via tensor analysis, he simply draws a space-time diagram, from which (with no hand waving at all), he describes the predicted results of the experiment in detail. He does not produce detailed numerical predictions in this chapter, though as he points out he easily could by including a little more metrical information in the diagrams.

My point, here, is not to suggest that space-time diagrams are in any way obscure or little-known tools of GR. They aren't. Everyone who has ever taken a course in GR has drawn a space-time diagram. What I am trying to emphasize, rather, is how broadly useful these diagrams can be, beyond the elementary and expository purposes to which they are often limited. These diagrams capture an immense amount of physics, in *any* physical situation to which it is appropriate to apply GR. And unlike many other visualization techniques in physics, such as Feynman diagrams or atomic level diagrams, space-time diagrams capture the essential geometrical properties of the physical configuration they represent. In other words, space-time diagrams actually support the kind of geometrical reasoning that they invite.

The present point is particularly useful in the context of some (old[1]) foundational questions in GR. One of the most striking features of Geroch's book, especially for a popular account of GR, is what expressions do *not* appear. (This is the second remark alluded to above.) The word "paradox" does not appear in the book; "paradoxical" appears once, on p. 146, in the context of insisting that there is nothing shocking associated with disagreements between observers concerning judgments of distance or length. Other expressions that one might expect to find in such a book, but which do not appear here, are "length contraction" and "time dilation". Geroch addresses the determinations of elapsed time, simultaneity, and length made by different observers in full calculational detail (here he does include numerical treatments). But he does so entirely in terms of space-time diagrams, from which perspective it simply does not make sense to talk about things like length contraction or time dilation as concrete physical phenomena. It is a mistake to approach GR by thinking of rods stretching or contracting, or of the gear wheels of a watch turning more or less slowly.[2]

The alternative view that Geroch recommends, the view suggested by taking space-time diagrams seriously, is one in which different observers make

[1] Such foundational problems are old mostly because the space-time perspective that Geroch advocates (or, perhaps, embodies) has been effective in resolving them. But one should not forget that many so-called "paradoxes" of GR greatly vexed physicists and philosophers in the early decades of the theory. They remain enshrined in most introductory texts on the subject.

[2] Emphasizing this way of thinking about the physics of GR is particularly salient, as Geroch's perspective is not entirely uncontroversial these days. Harvey Brown, in his recent book *Physical Relativity* (2005), seems to suggest that the dynamics of (for instance) rods stretching and contracting *are* crucial for understanding relativity theory.

different determinations of certain quantities or relations, such as length or simultaneity, by virtue of the structure of space-time and the measurement devices available to them. Take the well-known "twin paradox", wherein an astronaut leaves earth, travels for some distance at a high speed, and then returns to earth. When he lands, he is younger than his twin, who remained on earth the whole time.[3] Beginning with just a space-time diagram, one can easily determine how many ticks an earth-bound observer would attribute to an observer on a rocketship's clock (say), and vice-versa, without ever mentioning time dilation or proper time or anything of the sort. The two observers are represented by two different timelike curves in spacetime and they make determinations using various instruments. Using a space-time diagram, one can predict what these determinations will be. Often these observers' determinations will differ. But that is the end of the story: it should be no surprise, once the details of the measurements are spelled out diagrammatically, that they yield different results. There is simply no occasion for paradox to enter in. In fact, once one is accustomed to thinking about such problems geometrically, it is difficult to reconstruct what the paradox was supposed to be.[4, 5]

[3] Geroch does not treat this example by name; I bring it up only to show how on the geometric way of thinking, the paradox never has time to arise.

[4] Indeed, I do not think I have reconstructed the paradox here. I take it the difficulty is supposed to be something like as follows: from the perspective of the astronaut, earth recedes at a high speed for some time, and then changes directions and begins to approach again. And so, by some sort of symmetry principle, one is supposed to reason that the twin on earth ought to be younger. But they cannot both be younger than the other, and thus the paradox. Treating the problem geometrically, it is clear that there can be no such symmetry principle. The curves representing the two twins are not equivalent: they have different lengths.

[5] J. S. Bell (1987), in an essay called "How to teach special relativity" suggests another "paradox" of special relativity (often called the Bell Spaceship Paradox in his honor). One considers two spaceships initially drifting freely without any relative motion. The spaceships are assumed to be connected by a fragile string. At some mutually agreed upon time (this makes sense, since the spaceships initially have parallel inertial worldlines), both ships begin to accelerate uniformly and identically. The question is whether the string breaks. Bell reports an argument with a «distinguished experimental physicist» (Bell 1987, p. 68) at the Swiss accelerator laboratory CERN who believed that the string would *not* break. (Bell argued that it would.) To arbitrate, Bell and the experimentalist informally canvased the CERN theory division and discovered a consensus, at least initially, that the experimentalist was correct. Once the theorists spent time with the problem, however, they came to agree with Bell. Bell's own interpretation of these events is that many physicists have not recognized the physical importance of Lorentz-Fitzgerald length contraction. But an alternative interpretation, indirectly suggested by Geroch's book, is that Bell's colleagues at CERN would have done well to

The final remark I want to make concerning space-time diagrams and Geroch's presentation is this. One often sees space-time diagrams in introductory treatments of GR, but Geroch begins by developing the space-time diagram as a way of understanding classical space-times.[6] Treating classical theories from the perspective of space-time is much less common; as Geroch puts it, «in the Galilean view, space-time is a luxury; in relativity, a necessity» (p. 220). Yet this luxury is worth the indulgence. For one, treating classical theories, such as Newtonian theory (which has a Galilean space-time structure), in terms of four-dimensional space-time and space-time diagrams allows one to directly compare the mathematical structures of Newtonian and relativistic physics. Geroch develops classical space-times for just this purpose: he introduces relativistic space-times only after helping the reader to develop her everyday, classical intuitions in terms of space-time. Throughout the second part of the book, where he introduces GR, he reminds the reader of how to understand the differences between relativistic and classical space-times.

It is possible to go considerably further in developing Newtonian physics in terms of the geometrical structure of space-time than Geroch does, on account of the level at which he presents the material. In the early 1920s, in a lecture series at École Normale Supérieure, Élie Cartan recast Newtonian gravitation in the language of differential geometry. The resulting theory, now known as Newton-Cartan theory or "geometrized" Newtonian gravitation, is strikingly similar to GR: once again, the geometrical structure of spacetime depends on the distribution of mass within spacetime; conversely, gravitational effects are seen to be manifestations of the resulting geometry. It is possible to show that there is a rigorous sense in which Newton-Cartan theory is a limiting case of GR (where the limit consists in allowing the lightcone at every point to expand maximally). With the full Newton-Cartan theory in hand, one can extend

begin with a space-time diagram! Once one draws the appropriate diagram and considers the length of the string as determined by an observer co-moving with either of the string's ends, it is easy to see that the string stretches (and thus breaks).

[6] Geroch considers two kinds of classical space-times: Aristotelian space-time and Galilean space-time. An Aristotelian space-time, for Geroch, is one in which space-time has an absolute, fixed standard of rest and in which space has a fixed origin; a Galilean space-time is one in which all observers agree on determinations of simultaneity, but where there is no fixed standard of rest and thus no origin. For a more technical treatment of these and a variety of other classical space-times, see Earman (1989, ch. 2).

Geroch's comparative project and say precisely, in a wide variety of cases, how GR and classical physics relate to one another.[7]

Even without the fully geometrized gravitational structure, however, there is much to recommend looking at classical physics from the space-time perspective, especially to philosophers. Howard Stein (1967), for instance, has brought a geometrical, space-time understanding of classical physics to bear on historical questions concerning Newton, Leibniz, and Huygens' interpretations of Newton's theory. Earman (1989), meanwhile, surveys historical debates on absolute and relational theories of space from a firmly space-time perspective. Stein and Earman's work shows, I think, that the space-time perspective is as helpful in understanding and even resolving foundational problems in classical physics as it is in relativistic physics.

General Relativity from A to B is not intended as a philosophical work; nor is it meant as a text for specialists. And yet it serves as a potent reminder to both the physicist and the philosopher of physics of the power of a certain way of thinking about GR—and even classical physics—at both the calculational and foundational levels.

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[7] For more on Newton-Cartan Theory, the most systematic exposition available is Malament 2010. See also references therein for a sense of how the theory developed after Cartan's original presentation.

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